
2D Mass Mapping

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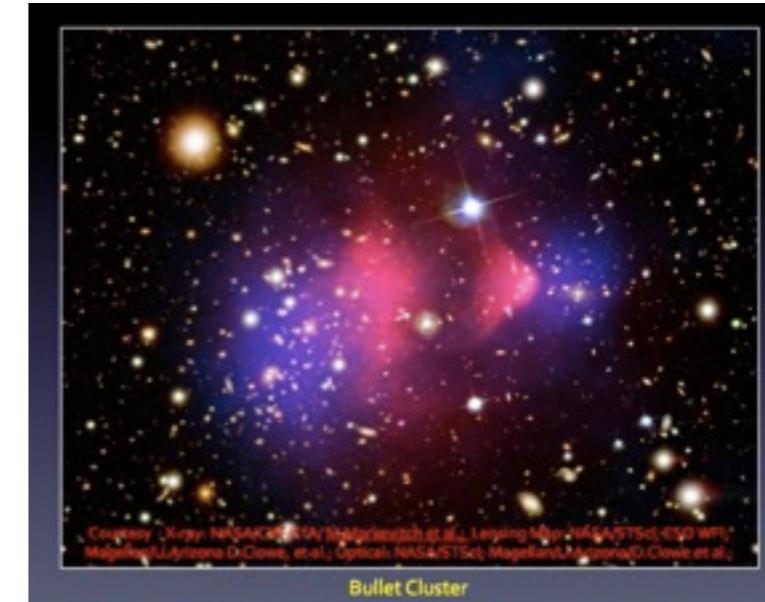
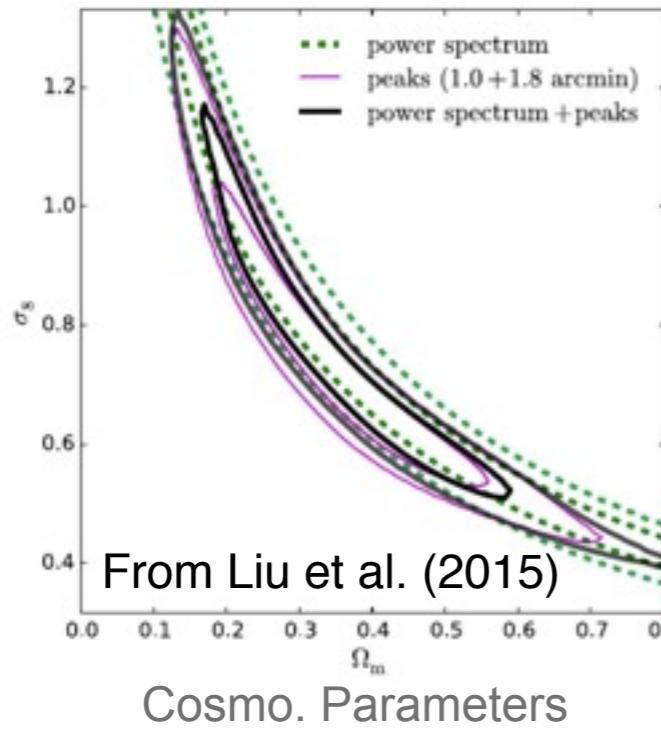
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Convergence Map

- **Mass Mapping: 2D-MASS-WL, 3D-MASS-WL**

- Originally, mass maps were considered not scientifically useful, but the situation is now clearly different.
- The field is evolving, and several 2D and 3D codes now exist.
- Science case work ongoing, and most requirements are not defined



$$\hat{\kappa} = P_1 \hat{\gamma}_1 + P_2 \hat{\gamma}_2$$

$$P_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$

$$P_2(k) = \frac{2k_1 k_2}{k^2}$$

=> PROBLEMS: Noise + Irregular Sampling

Mass mapping as an inverse problem

Binned data: $\gamma = F^* P F \kappa$

Unbinned data: $\gamma = T^* P F \kappa$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\min_{\kappa} \frac{1}{2} \| \gamma - \mathbf{P} \kappa \|_2^2 \quad \text{with} \quad \mathbf{P} = T^* P F$$

$$g = \frac{\gamma}{1 - \kappa} \longrightarrow$$

$$\min_{\kappa} \frac{1}{2} \| (1 - \kappa)g - \mathbf{P} \kappa \|_2^2$$

$L = \mathbf{T}^* \mathbf{P} \mathbf{F}$ is not directly invertible \Rightarrow **Linear inverse problem.**

Linear Methods:

- **Kaiser-Squires** (1993) + Gaussian smoothing

Non Linear methods:

- **For clusters:**
 - Model fitting algorithms (Bartelmann et al, 1996; Bradac et al, 2005; Jullo and Kneib, 2009).
 - Aperture Mass (Seitz and Schneider, 1996; 2002)
- **For larger fields:**
 - Maximum Likelihood (Bartelmann et al, 1996)
 - MemLens (Bridle et al, 1998; Marshal and Hobson, 2002)
 - **FastLens + MR-Lens** (Starck, Pires, Refregier, 2006; Pires et al, 2009)
 - **Glimpse2D** (Lanusse, Starck, Leonard, Pires, in preparation).

Aperture Mass and Wavelets



$$M_{ap}(\theta) = \int d^2\vartheta \gamma_t(\vartheta) Q(|\vartheta|)$$

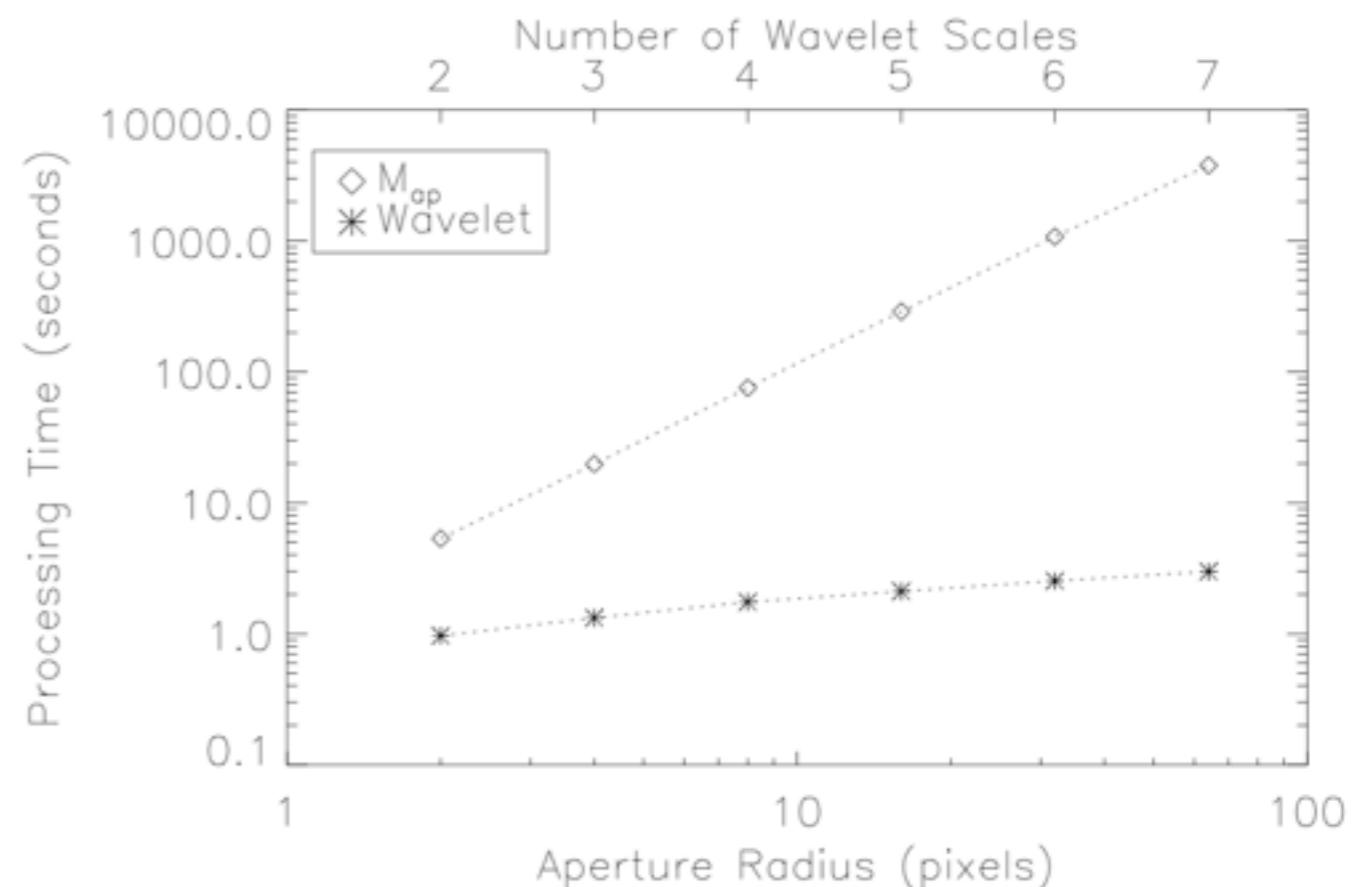
$$M_{ap}(\theta) = (\Phi^t \kappa)_\theta$$

⇒ Wavelets filters are formally identical to Mass aperture

A. Leonard, S. Pires, J.-L. Starck, ["Fast Calculation of the Weak Lensing Aperture Mass Statistic"](#), MNRAS, 423, pp 3405-3412, 2012.

but wavelets presents many advantages:

- compensated and **compact** support filters
- **fast** calculation:
- **all scales** processed in one step.
- **reconstruction** is possible
==> image restoration for peak counting



Sparsity and Mass mapping



Mass-Shear: $\gamma = \mathbf{P}\kappa$ with $\mathbf{P} = T^*PF$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\min_{\kappa} \frac{1}{2} \|\gamma - \mathbf{P}\kappa\|_2^2$$

sparse regularization

$$\min_{\kappa} \frac{1}{2} \|\gamma - \mathbf{P}\kappa\|_2^2 + \lambda \|\Phi^t \kappa\|_1$$

$$g = \frac{\gamma}{1 - \kappa}$$



$$\min_{\kappa} \frac{1}{2} \|(1 - \kappa)g - \mathbf{P}\kappa\|_2^2 + \lambda \|\Phi^t \kappa\|_1$$

=> Write the mass-mapping as a single optimization problem with a *multi-scale sparsity prior* addressing all these issues (i.e. reduced shear, missing data, noise).

The 2D Glimpse Algorithm

$$\min_{\kappa} \frac{1}{2} F(\kappa) + \lambda \|\Phi^t \kappa\|_1 \quad \text{with} \quad F(\kappa) = \frac{1}{2} \|(1 - \kappa)g - \mathbf{P}\kappa\|_2^2$$

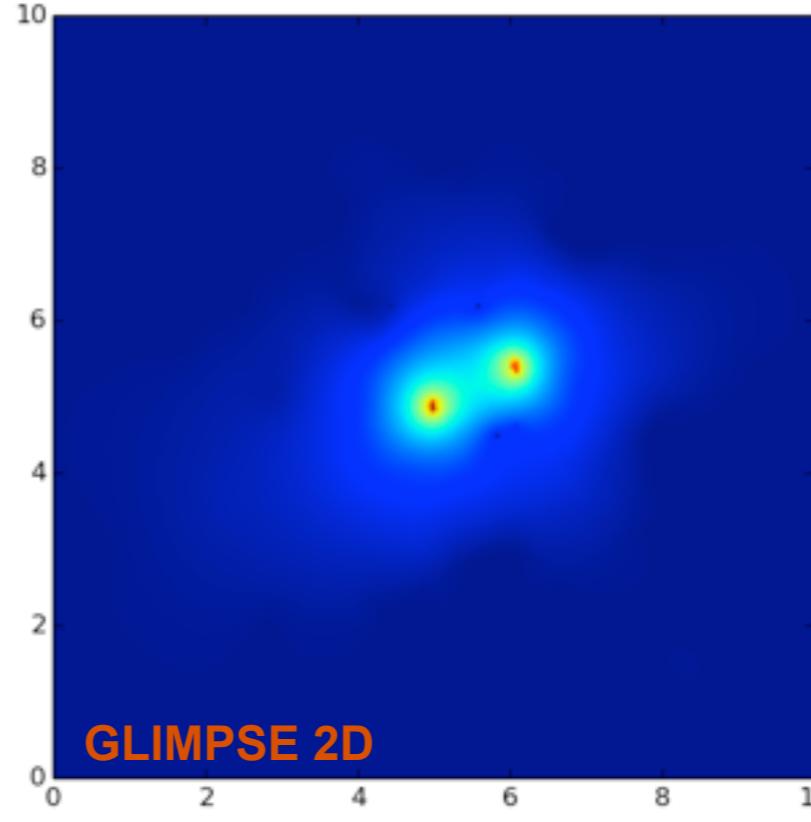
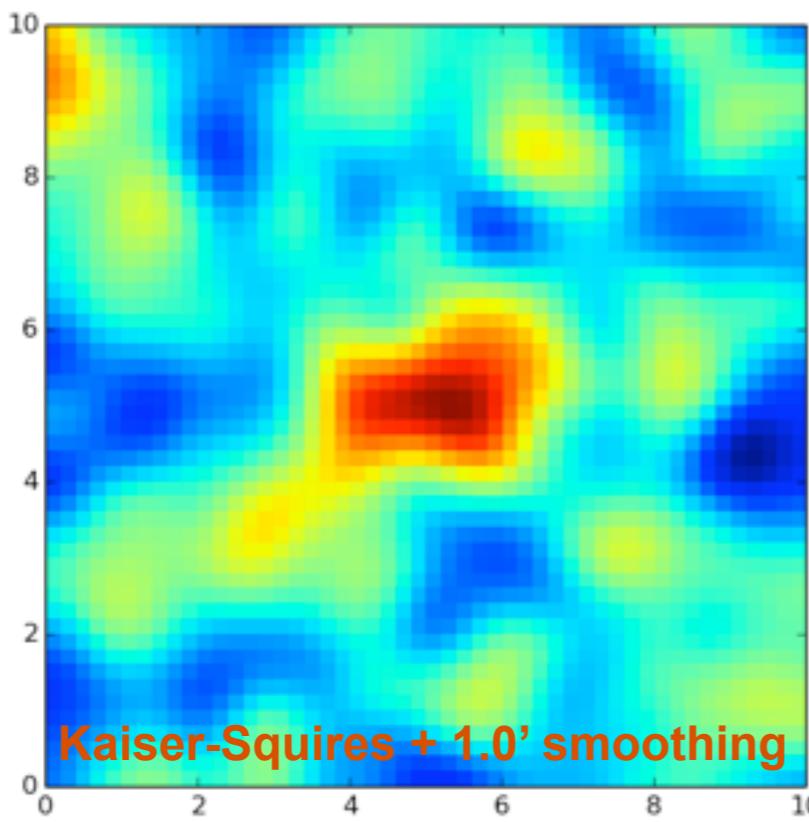
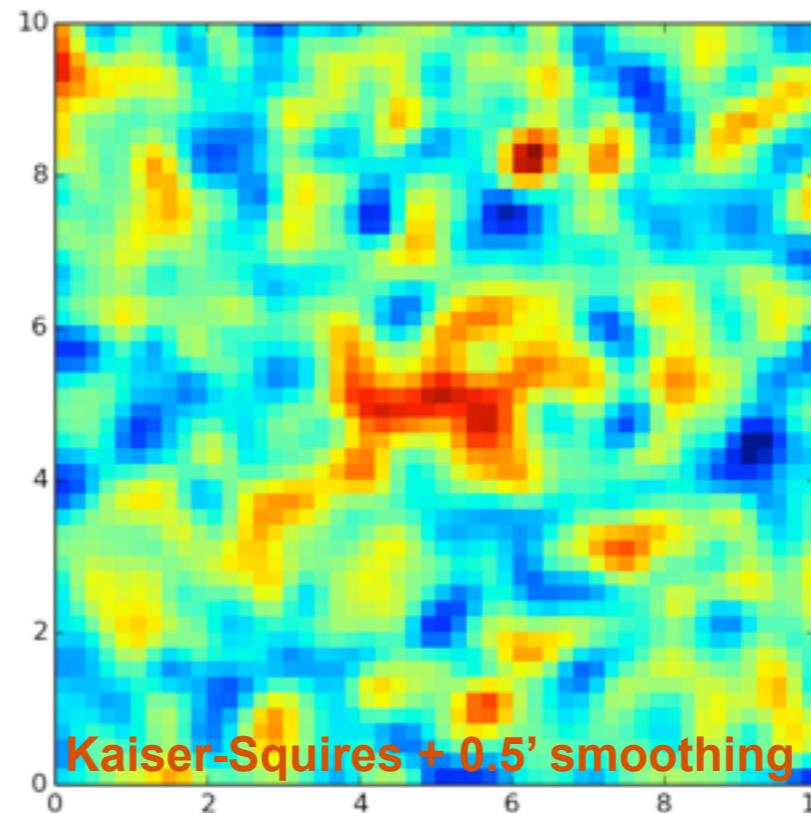
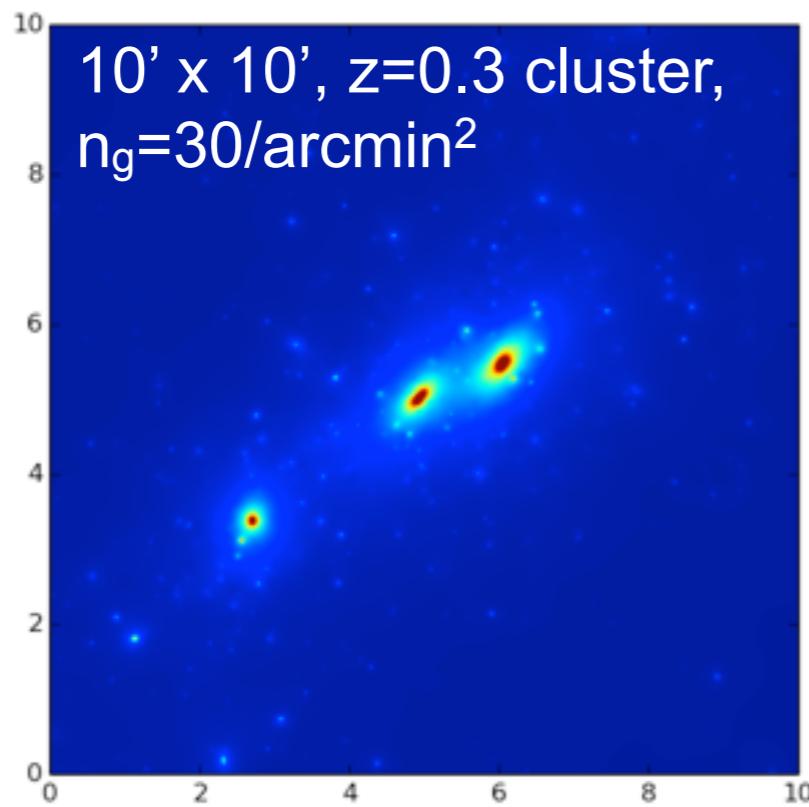
Primal-dual splitting:

$$\begin{cases} \kappa^{(n+1)} &= \kappa^{(n)} + \tau (\nabla F(\kappa^{(n)}) + \Phi \alpha^{(n)}) \\ \alpha^{(n+1)} &= (\text{Id} - \text{ST}_\lambda) (\alpha^{(n+1)} + \Phi^t (2\kappa^{(n+1)} - \kappa^{(n)})) \end{cases}$$

adapted from Vu (2013)

- Fast and flexible algorithm
- Sparsity constraint λ estimated locally by noise simulations \implies Accounts for **survey geometry, varying noise levels**

Missing Data + Noise



Flexion

Shear is noise dominated on small scales ==> Substructures are lost

Small-scale substructure can be recovered from strong lensing when available.

Gravitational **Flexion** is useful in the intermediate regime.

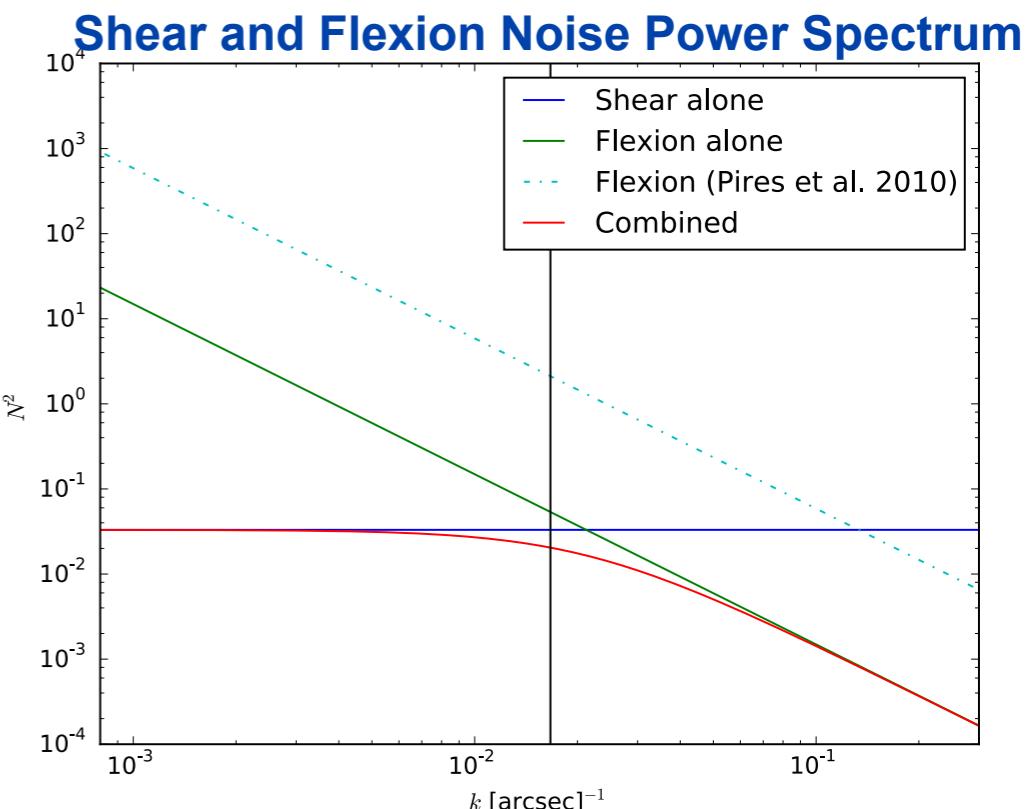
Flexion gives information relative to the third order derivatives of the lensing potential



Shear (left) and first flexion (right) (Bartelmann 2010)

$$\mathcal{F} = \nabla \kappa$$

$$F = \frac{\mathcal{F}}{1 - \kappa}$$



Flexion Information

We can integrate flexion in our reconstruction framework

=> **Jointly** fit shear and flexion

$$\min_{\kappa} \frac{1}{2} \| (1 - \kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \|_2^2 + \lambda \| \Phi^t \kappa \|_1$$

=> **Jointly** fit shear and flexion with redshift information

$$\min_{\kappa} \frac{1}{2} \| (1 - \mathbf{Z}\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \mathbf{Z} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \|_2^2 + \lambda \| \Phi^t \kappa \|_1$$

with $\mathbf{Z} = \Sigma_{critic}^\infty / \Sigma_{critic}(z_i)$

$$\Sigma_{crit}^\infty = \lim_{z \rightarrow \infty} \Sigma_{crit}(z)$$

$$\Sigma_{crit}(z_s) = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

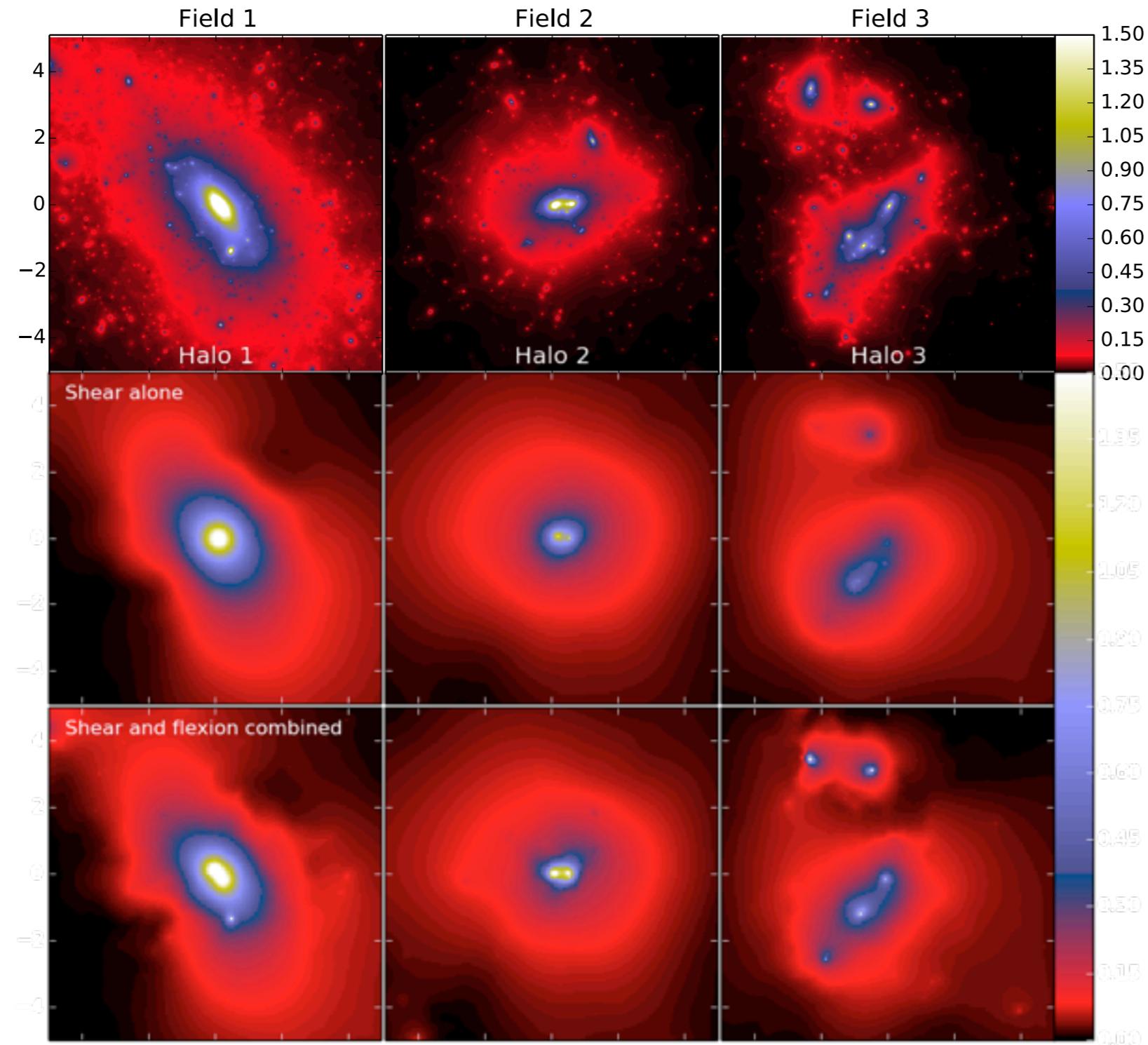
Individual redshifts have two benefits:

- Directly map the **surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac et al. 2004)

Simulations with Flexion

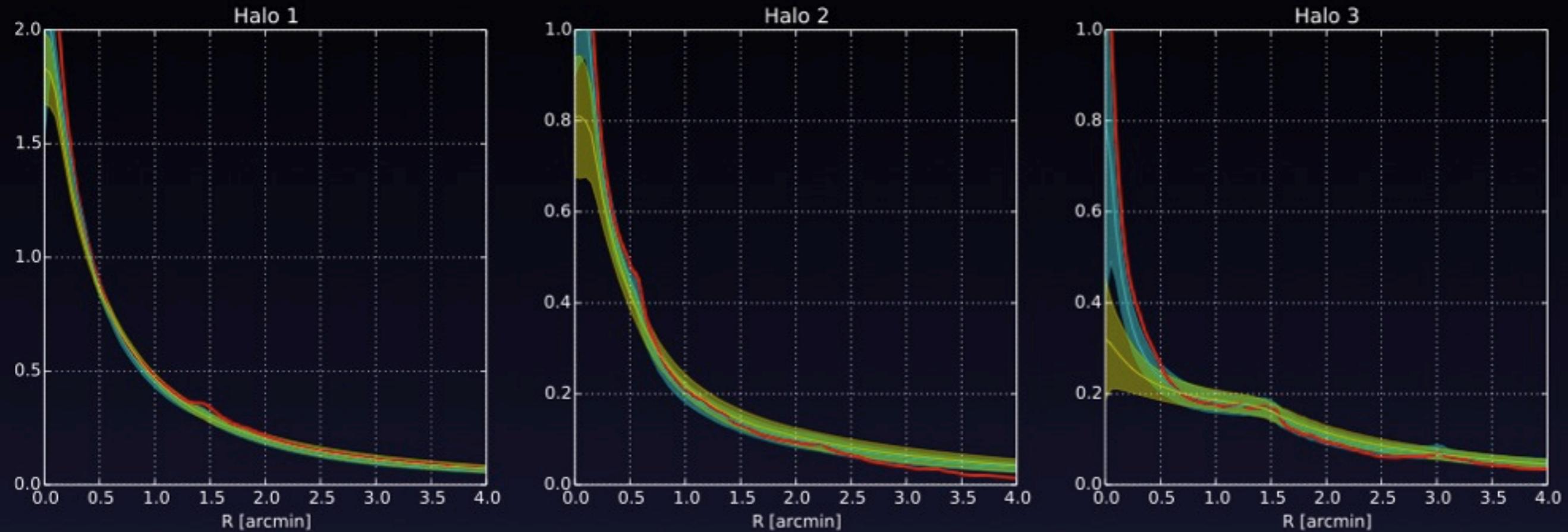
Simulate reduced flexion

Flexion noise $\sigma_F = 0.029 \text{ arcsec}^{-1}$ (Cain et al, 2011)



Reconstruction from one realisation

CosmoStat Lab



Benefits of adding flexion:

- Improvement on the recovered profiles below 0.5 arcmin
- Recovery of small-scale substructure at the 10 arcsec scale

Conclusions

* **GLIMPSE2D**: A new mass mapping algorithm, based on sparsity and proximal optimization theory:

- Does not require angular binning of the ellipticities
- Accounts for reduced shear
- Proper regularization of missing data

A new framework :

- => Can include individual redshift PDFs of sources
 - Directly map the **surface mass density** of the lens
 - Mitigate the **mass-sheet degeneracy** when the convergence becomes significant (Bradac et al. 2004)
- => Can include flexion measurements if available
- => Can be also be used for non-parametric high-resolution cluster density mapping from weak lensing alone

Lanusse F., Starck J.-L., Leonard A., and Pires S. (2015), High Resolution Weak Lensing Mass Mapping combining Shear and Flexion , in prep.

- **Bridge** between **low resolution** weak lensing and high resolution **strong** lensing
- Can recover cluster **substructures** without strong lensing information
- Ideal tool for investigating **models of dark matter**

* **The science case is not yet mature: no requirement.**