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Density PDF and density profile in low-density regions:

an alternative probe for Euclid era cosmology?

Bernardeau, Pichon, Codis: arXiv : 1310.8134

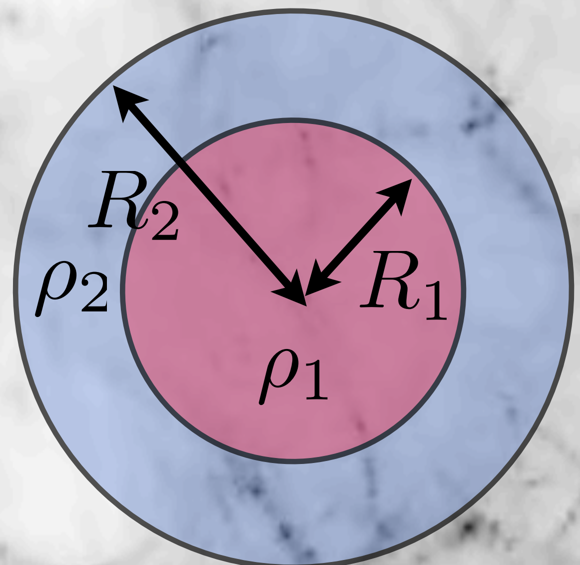
Journées Euclid France, 5 Dec. 2013

Messages to bring back home:

➢ we are able to ***predict very accurately N-pt statistics in the non-linear regime*** using Count-In-Cells statistics : low-redshift observables have analytical and cosmology-dependent predictions e.g 1% on P(ρ) @ z=0.7

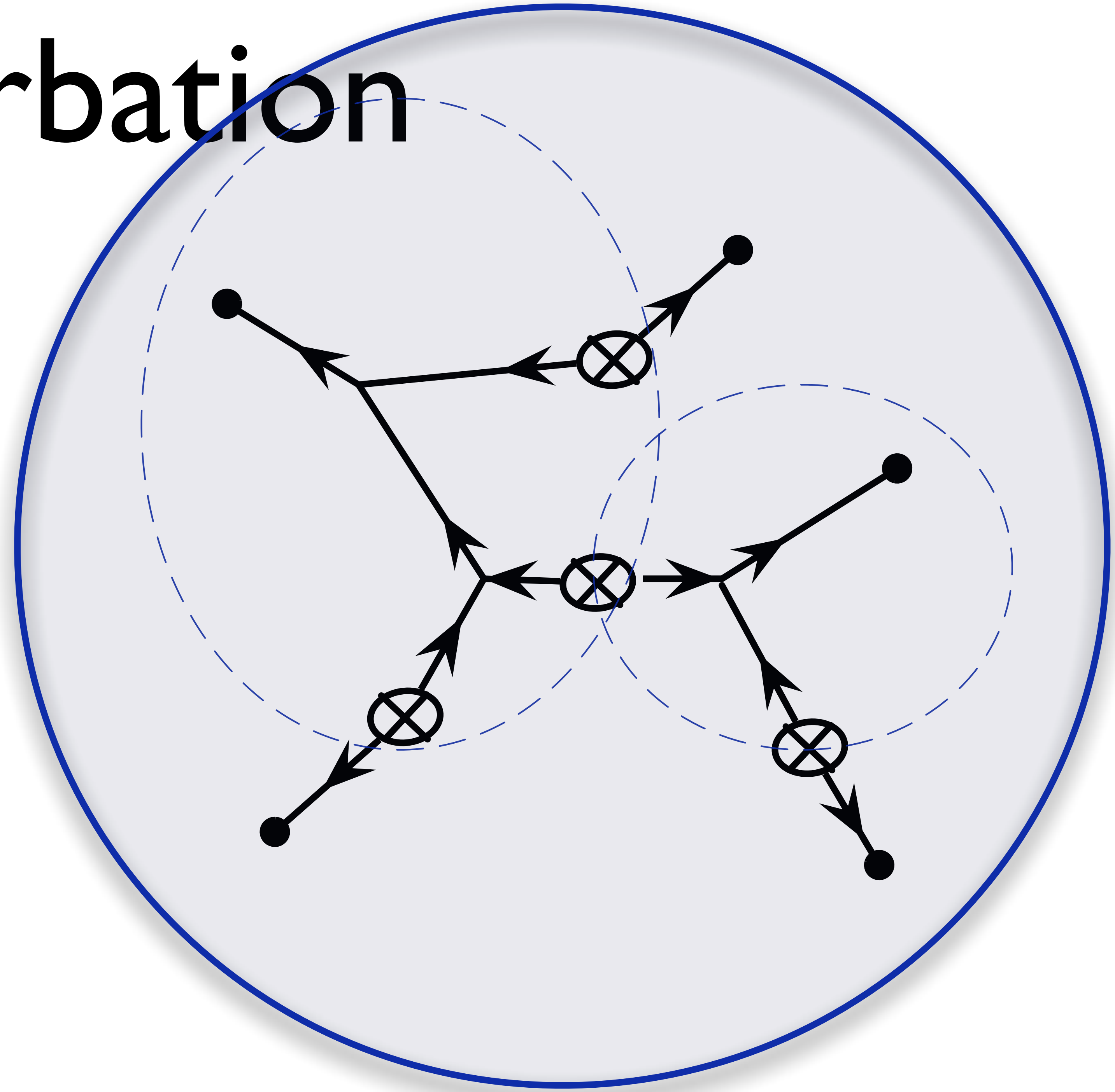
➢ at tree order, everything is encoded in the dynamics of the ***spherical collapse***

➢ we are able to do the theory of the *slope* of the density field:
Cosmic scatter is reduced in low-density regions motivating the study of ***void profiles.***



$$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1}$$

Introduction : Basics of perturbation theory



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A self-gravitating expanding dust fluid

The Vlasov-Poisson equations (collision-less Boltzmann equation) - f(x,p) is the phase space density distribution - are fully nonlinear.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\frac{\partial}{\partial \mathbf{x}}\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \bar{n}\right)$$

The rules of the game:

➢ **single flow** equations

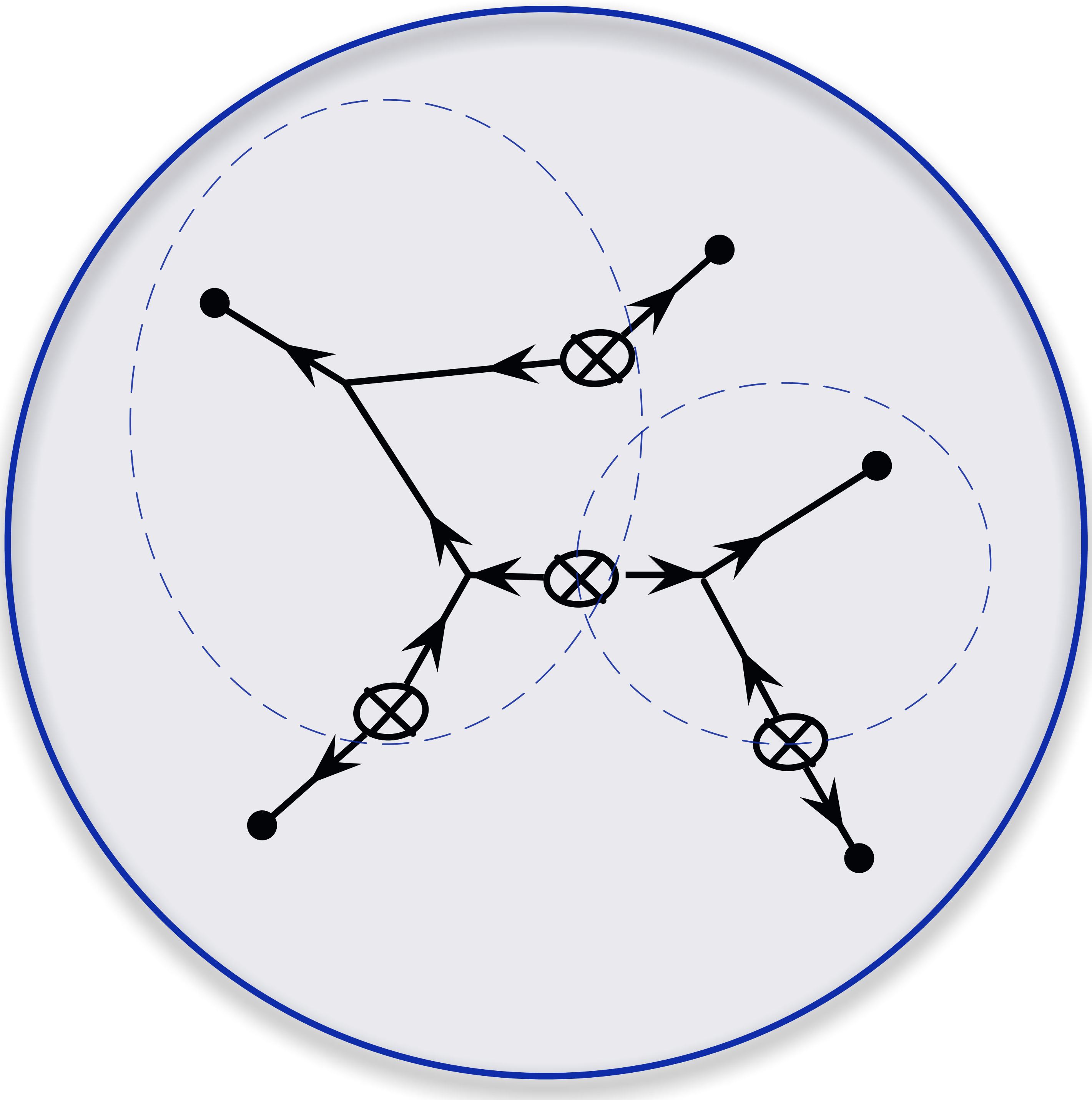
Peebles 1980; Fry 1984; Bernardeau, Colombi, Gaztañaga, Scoccimarro, 2002

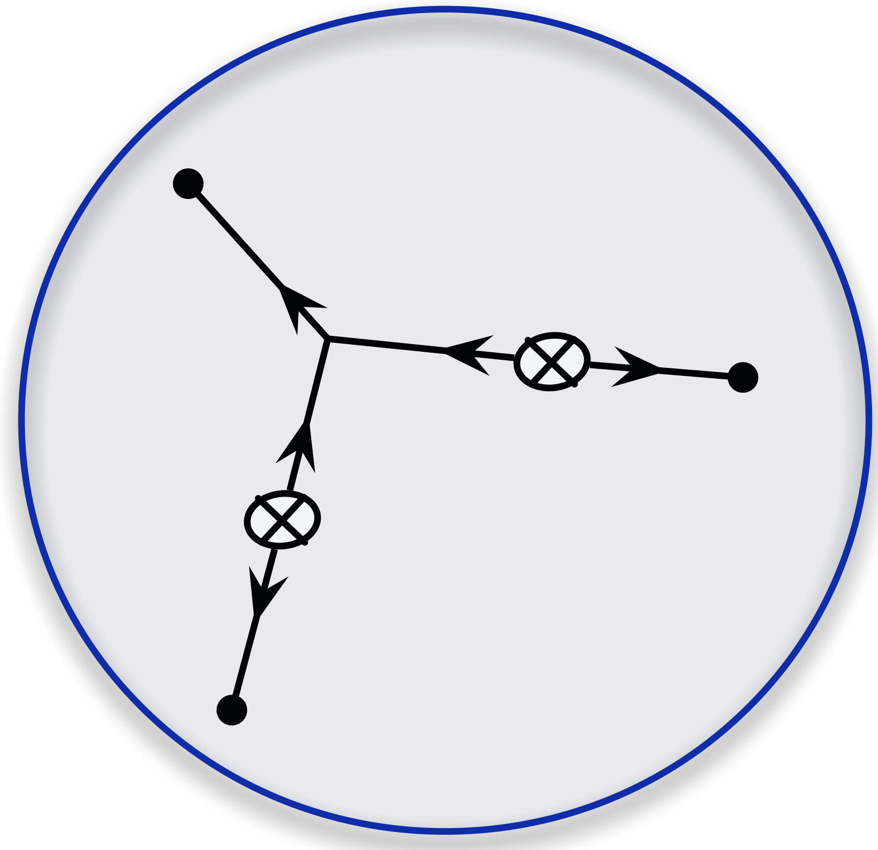
$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t) + \frac{1}{a}[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)]_{,i} &= 0 \\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t) + \frac{\dot{a}}{a}\mathbf{u}_i(\mathbf{x},t) + \frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\Phi_{,i}(\mathbf{x},t) + \cancel{\ldots} \\
\Phi_{,ii}(\mathbf{x},t) - 4\pi G\overline{\rho}\; a^2\; \delta(\mathbf{x},t) &= 0
\end{aligned}$$

➢ ***it is possible to analytically expand the cosmic fields with respect to initial density fields***

$$\delta(\mathbf{x},t) = \delta^{(1)}(\mathbf{x},t) + \delta^{(2)}(\mathbf{x},t) + \ldots$$

Example of contribution to the 3 and 5-point correlation functions at tree order





it has a non-trivial dependence on the wave vectors through the functions F3 and F2

Charting PT

number of loops in standard PT for Gaussian Initial Conditions

Order of observable in field expansion

	Tree order LO	1-loop NLO	2-loops NNLO	2.5 loops	3-loops	...p-loops
2-point statistics	OK	OK	OK	EFT	partial exact results	partial resum
3-point statistics	OK	OK (but not systematics)				partial resummations
4-point statistics	OK	to be done... (cosmic variance)				
N-point statistics	OK, in specific geometries (counts in cells)					

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breaks down for smaller σ^2

The trick of the spherical collapse leads to analytic predictions in the non-linear regime @ few percent level until $\sigma^2 \sim 0.7$!!

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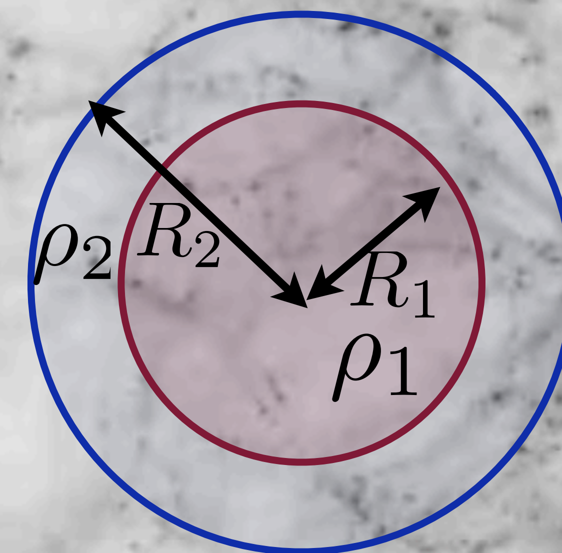
Density PDFs in concentric cells



description of full joint PDF densities in concentric cells:

$$P(\rho(R_1), \rho(R_2))\, \mathrm{d}\rho(R_1)\, \mathrm{d}\rho(R_2)$$

ρ_1 ρ_2

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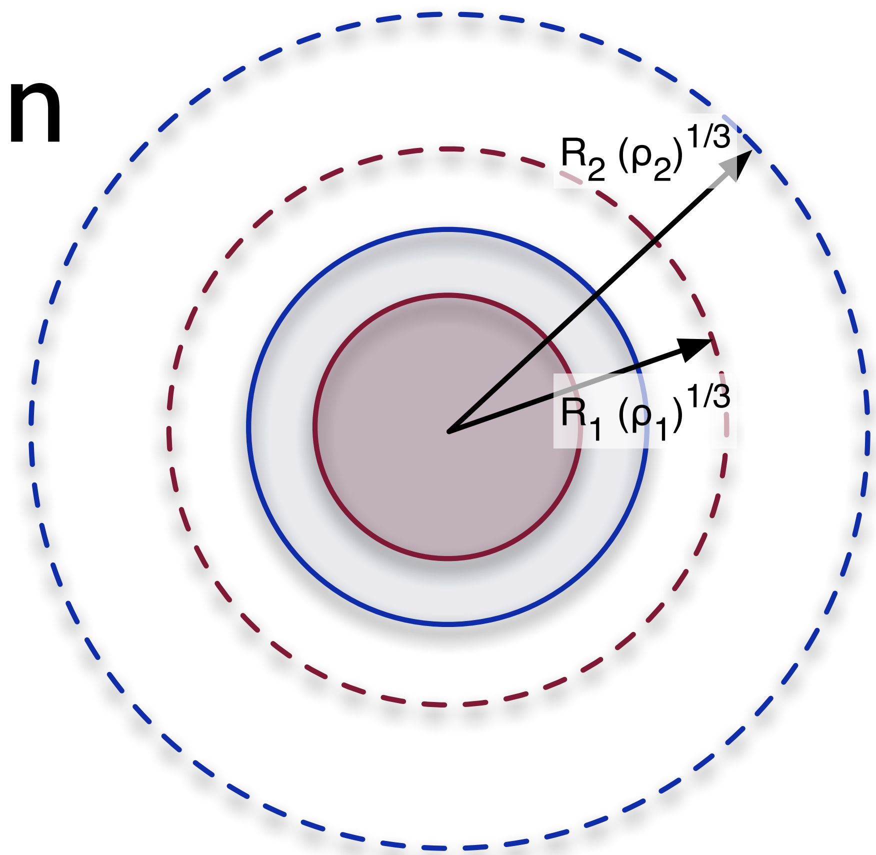
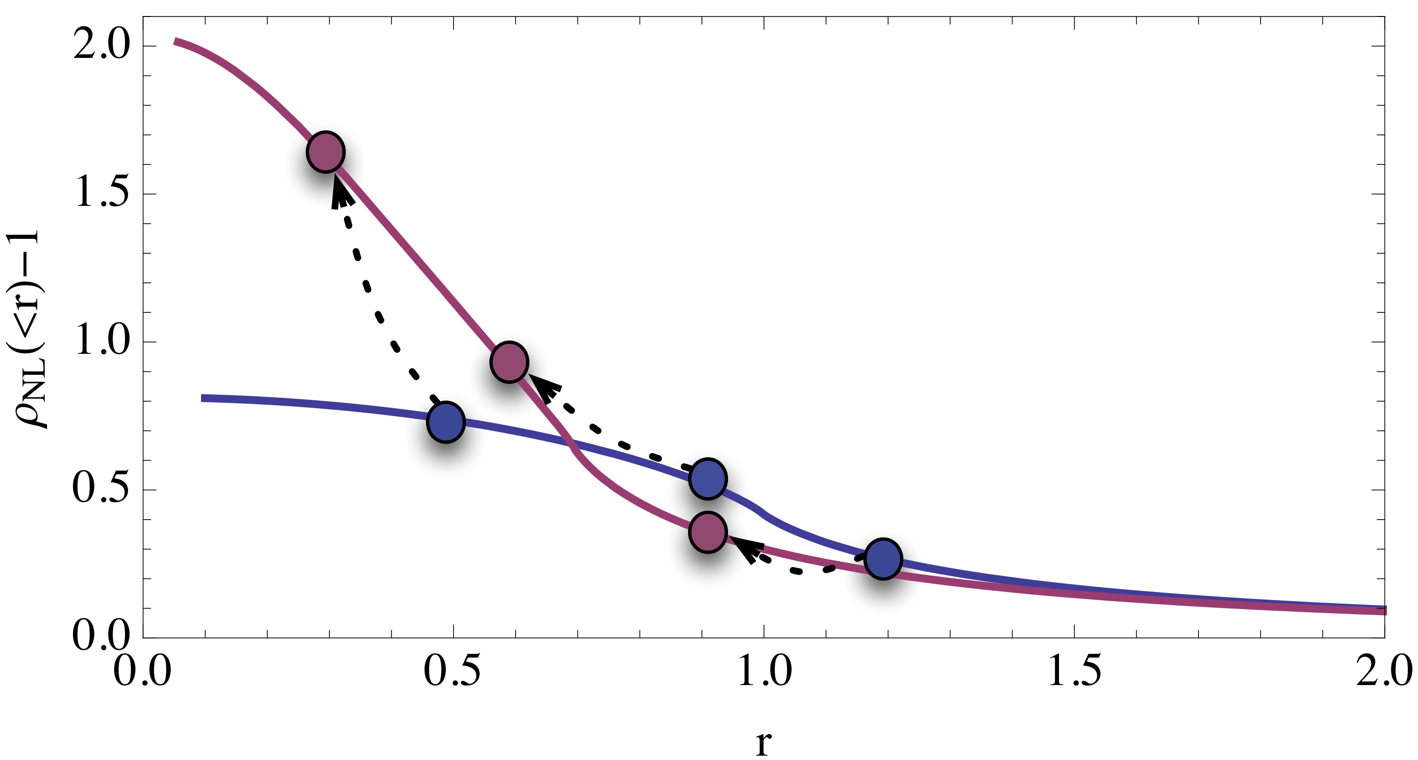
The spherical collapse: the solution for specific initial conditions



The radius evolution

$$\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} = -\frac{GM(<R)}{R^2}$$



The exact non-linear mapping for spherically symmetric initial field (for growing mode setting)

For spherical symmetry perturbations there exists a function ζ that gives the density at time η knowing the density ρ_0 within the same Lagrangian radius at time η_0,

$$\zeta_\rho(\eta, \rho_0, \eta_0)$$

cosmology-dependent!

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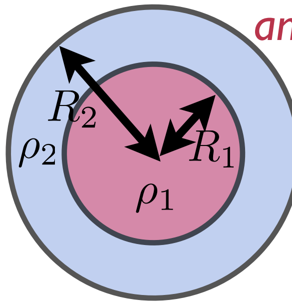
The mathematical part, construction of the whole cumulant generating function

from ideas in Bernardeau' 94
see also Bernardeau & Valageas '00
and fully developed in Valageas '02



It is given by the following relation (multi-dimensional Laplace transform of joint-PDFs) :

$$\varphi(\{\lambda_k\}) = \sum_{p_i=0}^{\infty} \langle \Pi_i \rho_i^{p_i} \rangle_c \frac{\Pi_i \lambda_i^{p_i}}{\Pi_i p_i!} \simeq \lambda_i \langle \rho_i \rangle + \lambda_i \lambda_j \langle \rho_i \rho_j \rangle + \dots$$

$$\exp\left[\varphi(\{\lambda_k\})\right] = \mathcal{M}(\{\lambda_k\}) = \left\langle \exp(\sum_i \lambda_i \rho_i) \right\rangle$$

$$= \int_0^{\infty} \prod_i \mathrm{d}\rho_i P(\{\rho_k\}) \exp\left(\sum_i \lambda_i \rho_i\right)$$

initial density contrast

Formal solution :

$$\exp\left[\varphi(\{\lambda_i\})\right] = \int \mathcal{D}\left[\tau(\vec{x})\right] \mathcal{P}\left[\tau(\vec{x})\right] \exp(\lambda_i \rho_i \left[\tau(\vec{x})\right])$$

known Gaussian pdf involving the linear power spectrum

Principle of the calculations : in the small variance approximation one can look for the most probable configuration - for fixed ρ_i - and compute the resulting cumulant generating function using the steepest-descent method.

The (conjectured) solution for spherical cells : an initial spherical perturbation the profile of which can be computed from ***spherical collapse*** solution.

$$\rho_i = \zeta_{\mathrm{SC}}(\tau_i)$$

one-to-one mapping

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Application 1: 1-cell PDF

The inverse Laplace transform, $$\mathcal{P}(\hat{\rho}_1) = \int_{-i\infty}^{+i\infty} \frac{d\lambda_1}{2\pi i} \exp(-\lambda_1 \hat{\rho}_1 + \varphi(\lambda_1))$$

requires integration into the complex plane.

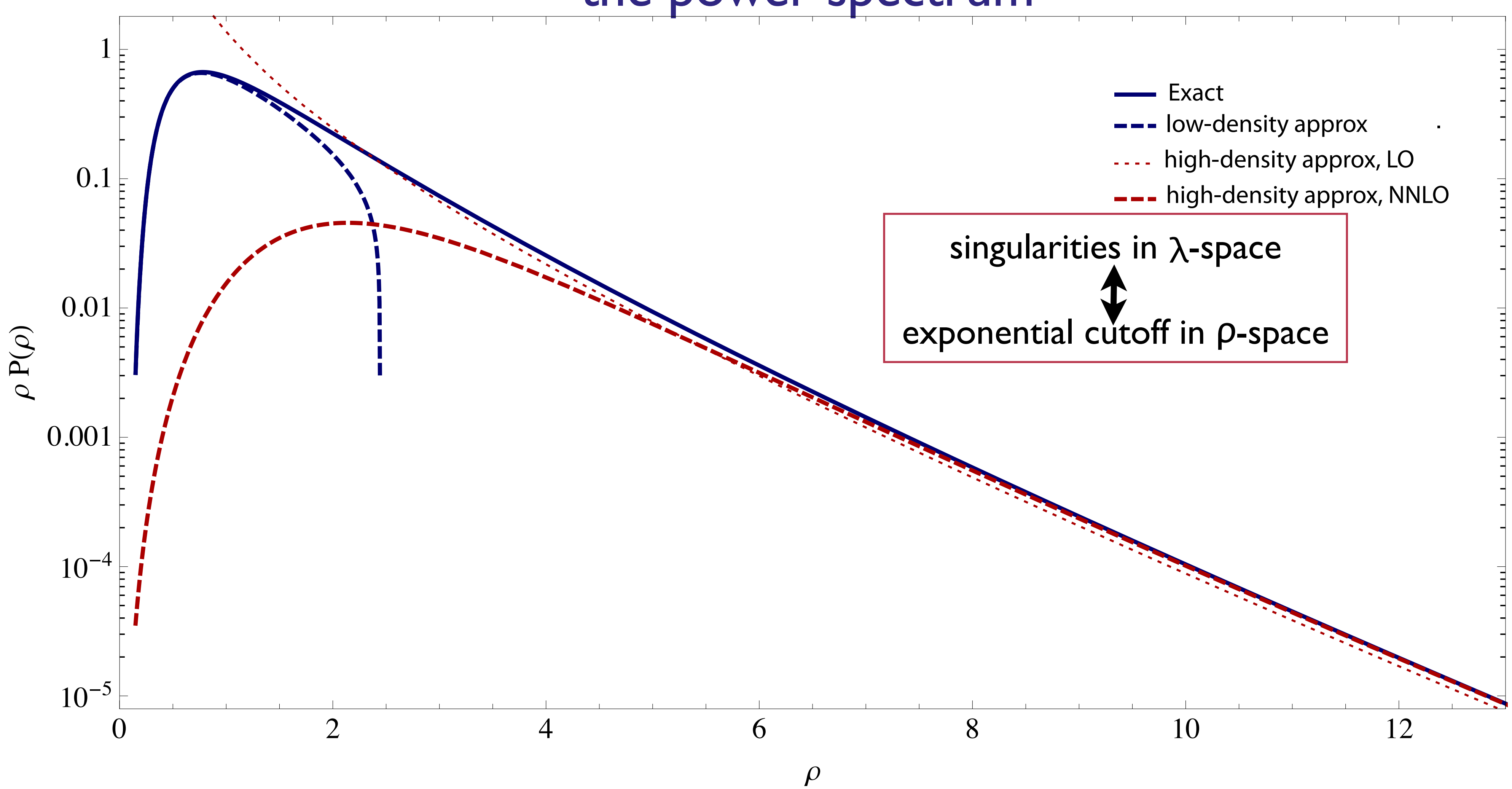
$$P(\rho) = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\partial^2 \Psi(\rho)}{\partial \rho^2}} \exp\left[-\Psi(\rho)\right]$$

low-density approximation

$$P(\rho) = \frac{3a_{\frac{3}{2}}}{4\sqrt{\pi}} \exp\left(\varphi^{(c)} - \lambda^{(c)}\rho\right) \frac{1}{(\rho + r_1 + r_2/\rho + \ldots)^{5/2}};$$

large-density approximation

functions of the cosmology via the power spectrum



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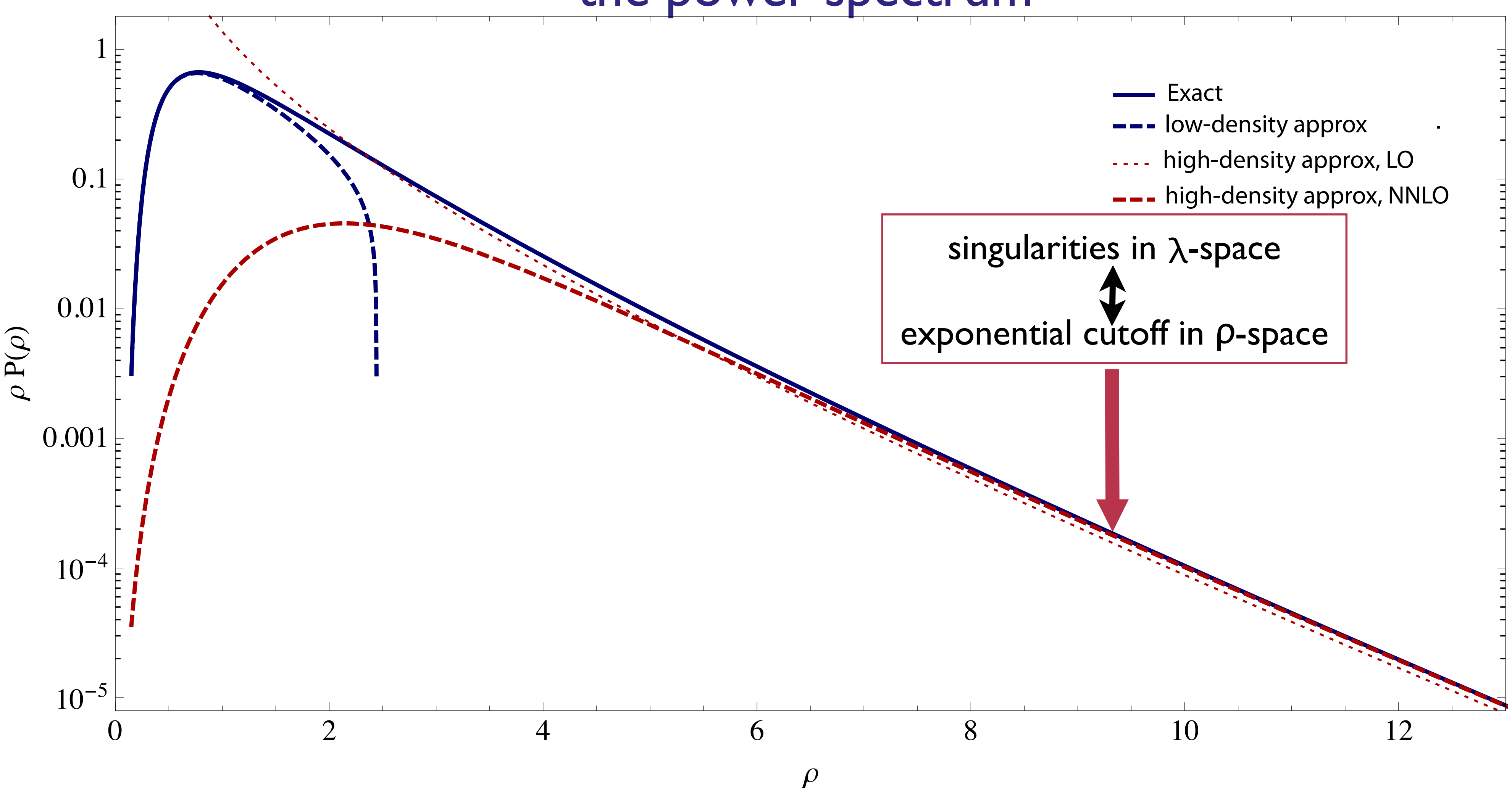

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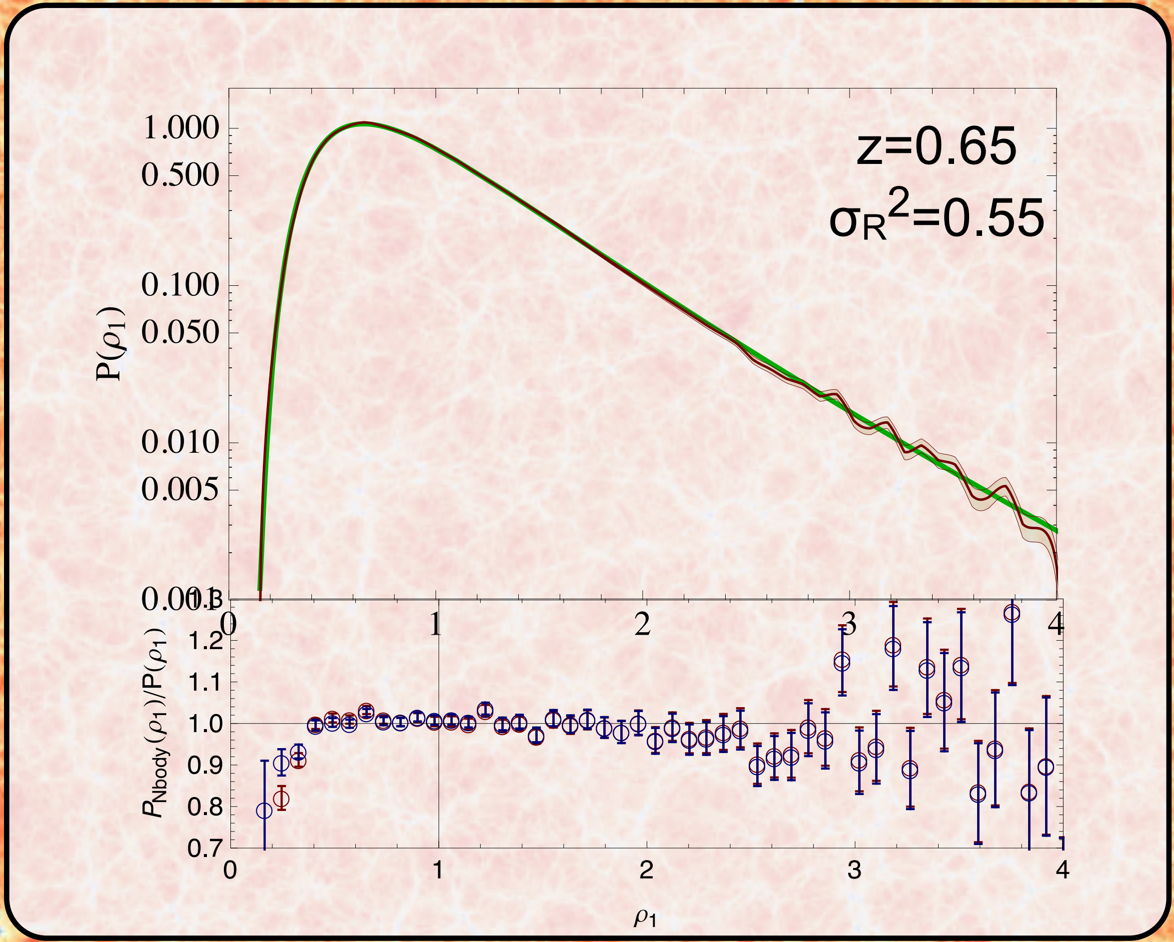
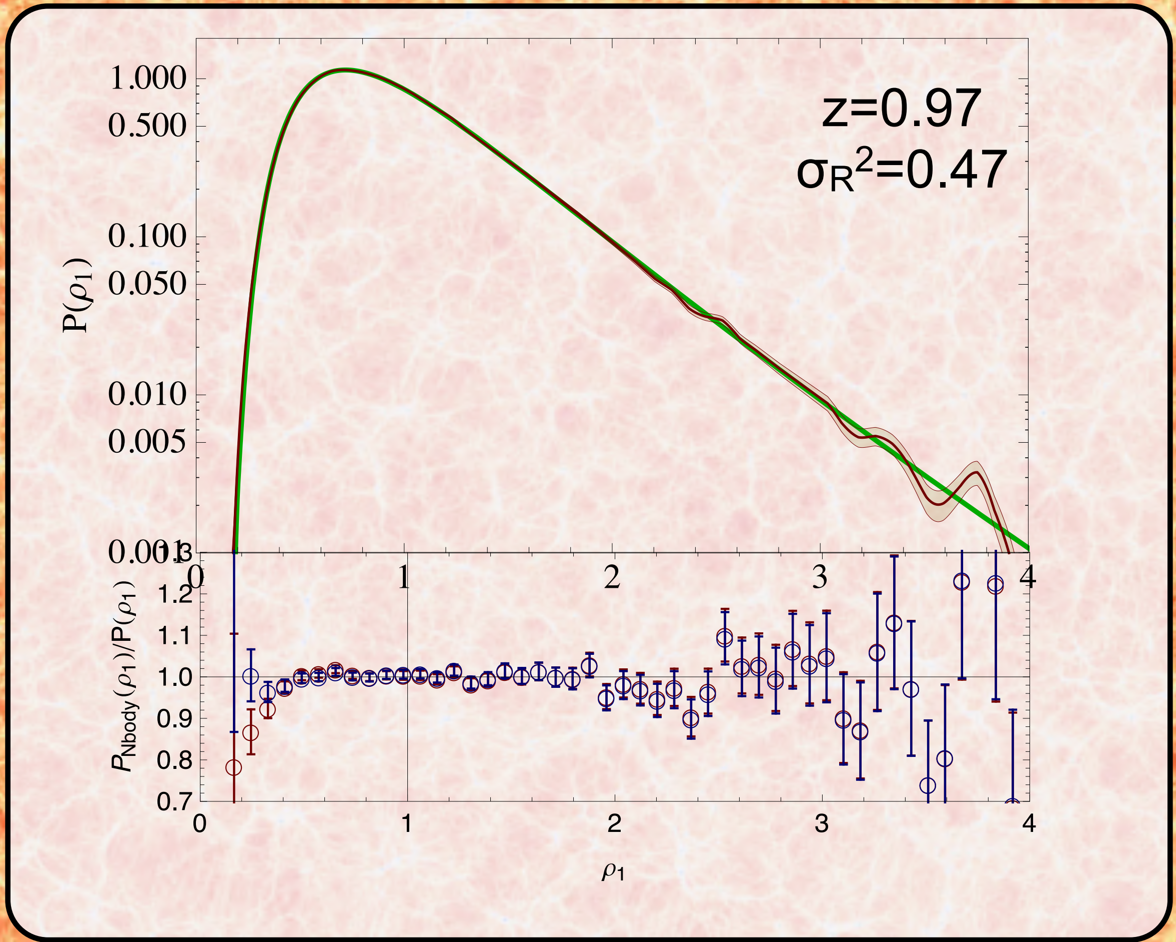
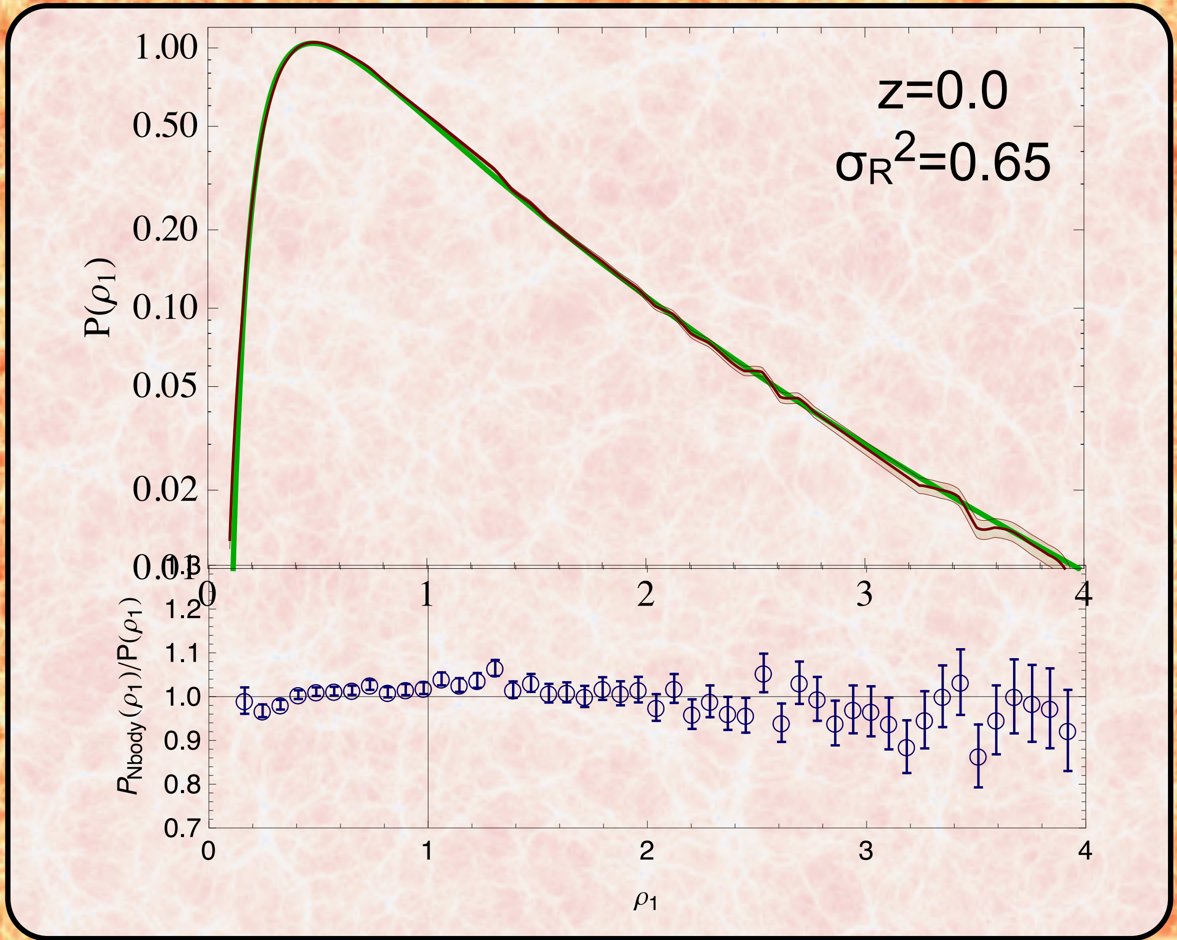
Comparison with simulations: the 1-cell PDF

(500 h^{-1} Mpc)^3
$R = 10\ h^{-1}$ *Mpc*

agreement even deeply in the non-linear regime, in the rare event tails of the PDF!







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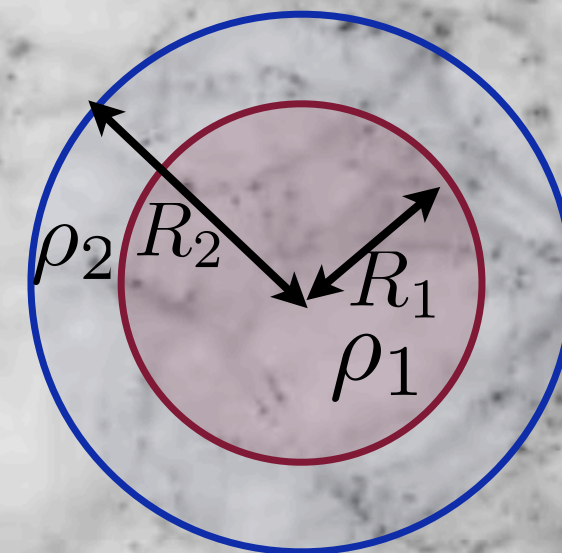
Application 2 (two cells): Density slopes and profiles



$$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1}$$

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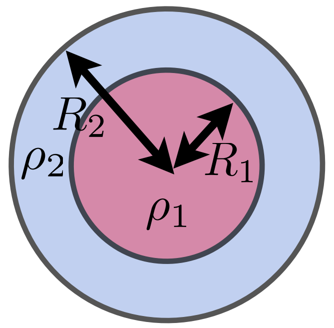
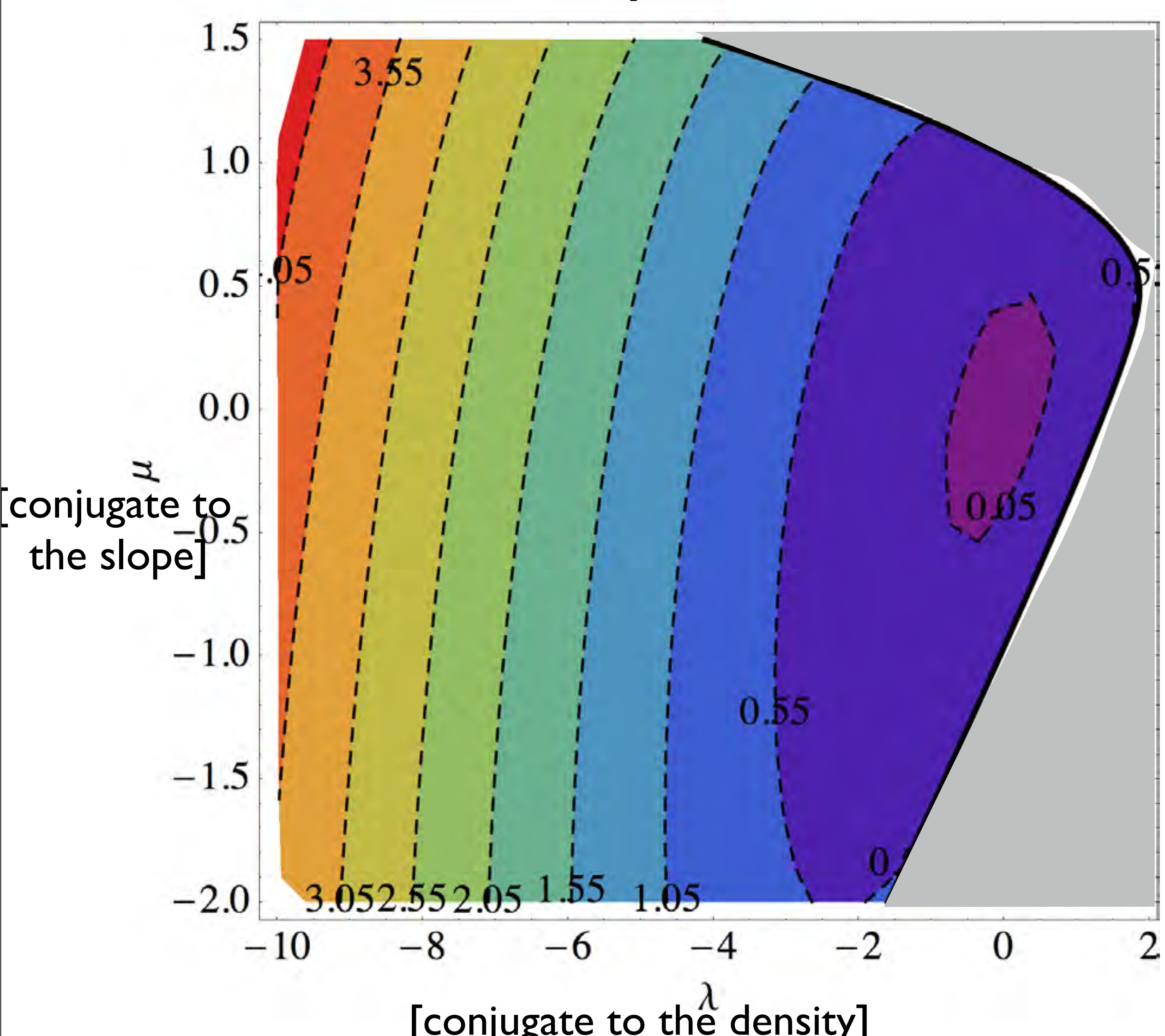
The 2-cell cumulant generating function



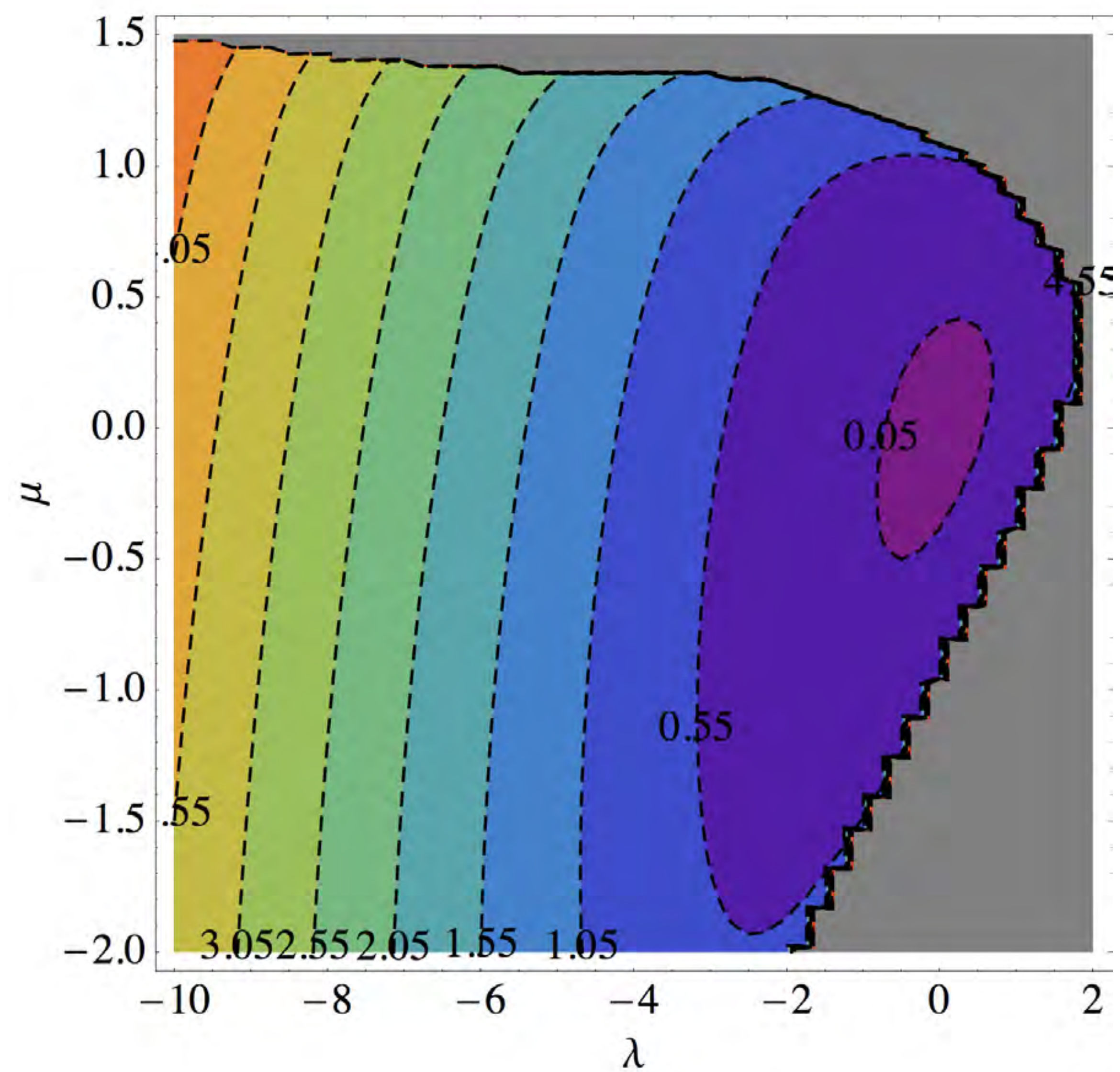
The density slope : $$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1}$$

The global shape of the joint cumulant generating function of the density slope, s, with the density ρ_1 (an observable itself):

theory



numerical results for σ = 0.51



critical lines = stationary constraint is singular / signal to noise > 10%

$$\langle \exp(\lambda_1 \rho_1 + \lambda_2 \rho_2) \rangle \to \infty$$

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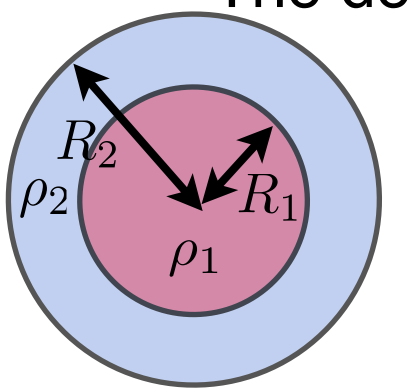
The PDF of density slope

see also Bernardeau & Valageas '00

The density slope :

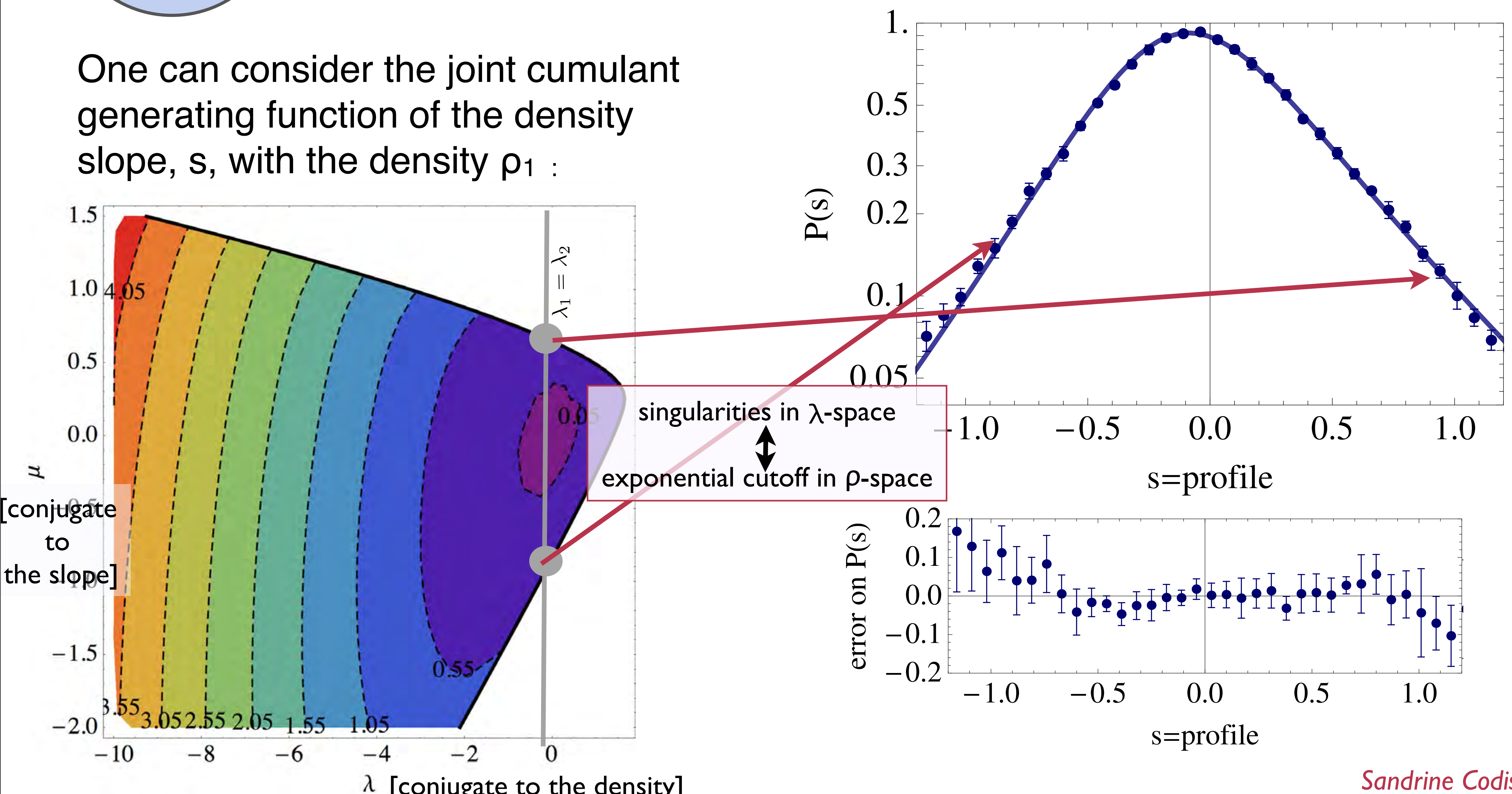


$$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1}$$

Similarly to the 1-cell density PDF one can then compute the one-point density profile PDF.

$$P(s) = \int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\lambda_2}{2\pi i} \exp\left[-\lambda_2 s + \varphi(-\lambda_2, \lambda_2)\right]$$

One can consider the joint cumulant generating function of the density slope, s, with the density ρ_1 :



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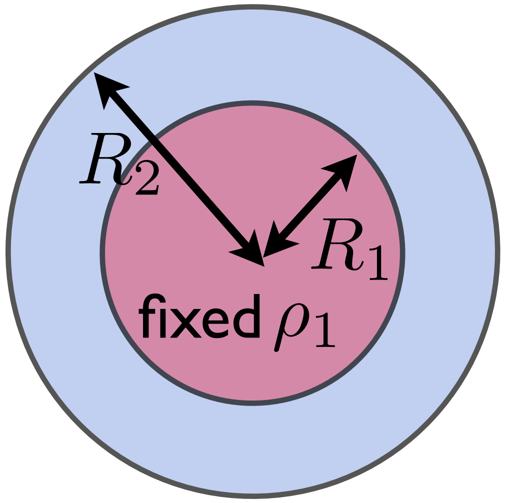
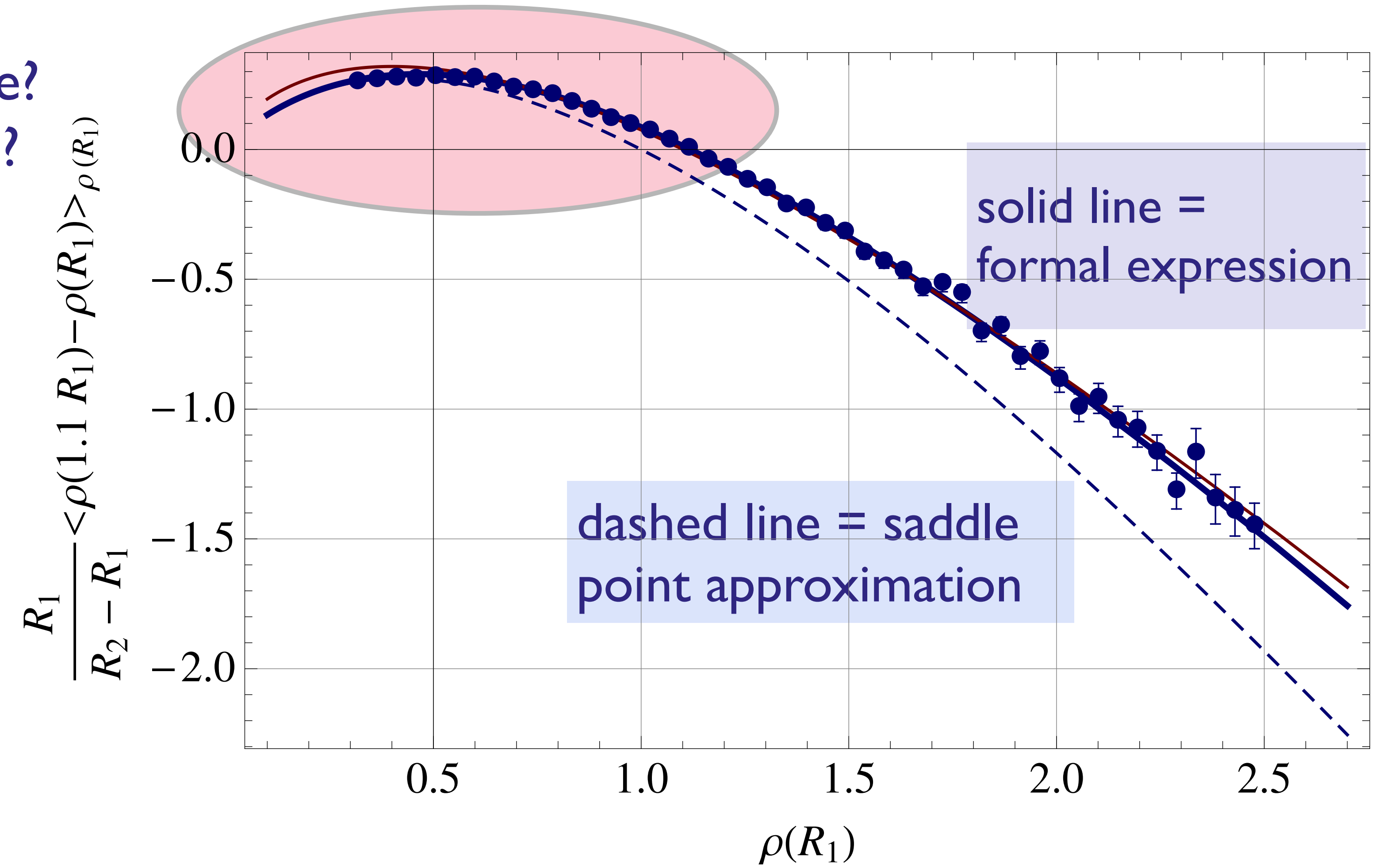
The expected slope given a density constraint

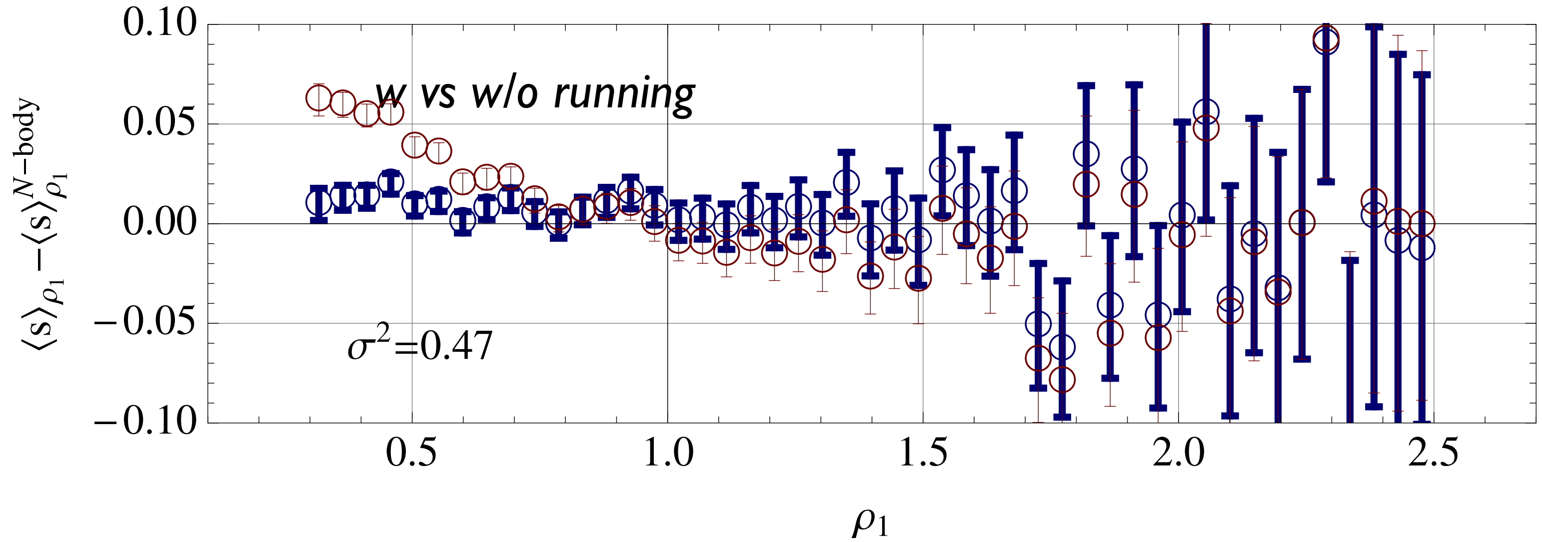


slope?
expectation value?
cosmic variance?



Best predictions are in the low-density regime. This is where saddle-point corrections are less important.

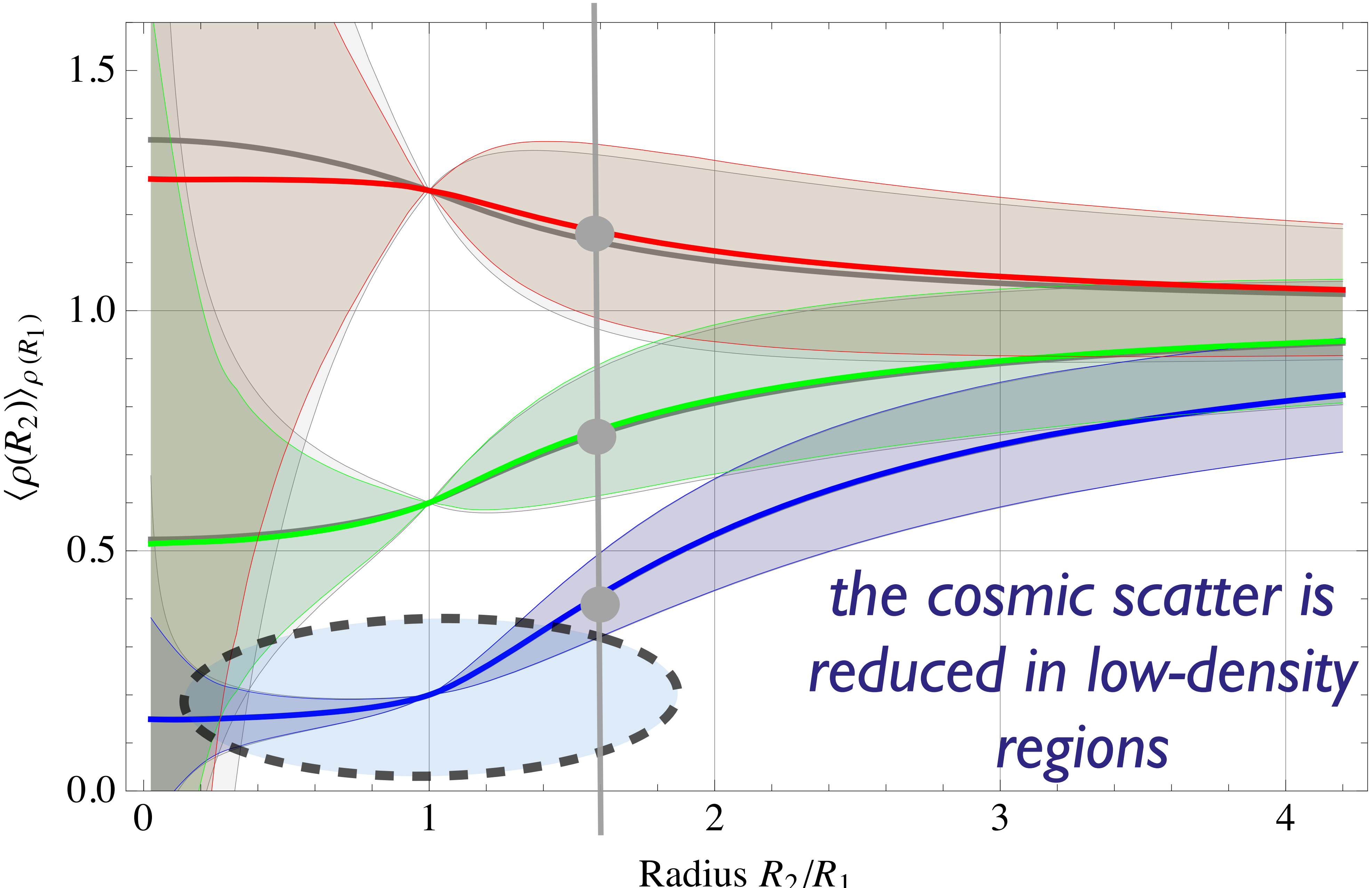
Impact of scale dependence of the power spectrum index

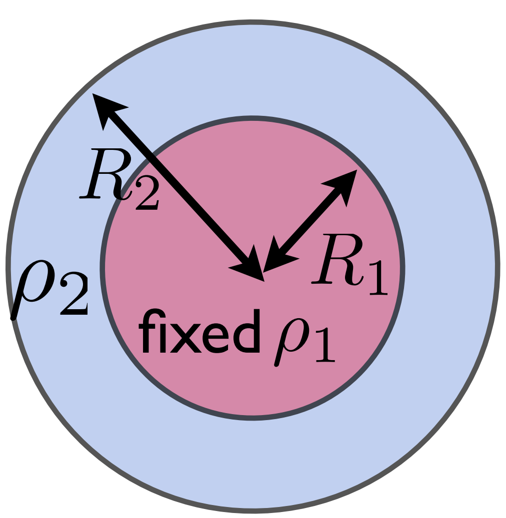


The profile shape

(expected density as a function of the radius)

Expected density as a function of radius given a constraint at a given scale.





ρ_2 ?
expectation value?
cosmic variance?
as a function of R_2?

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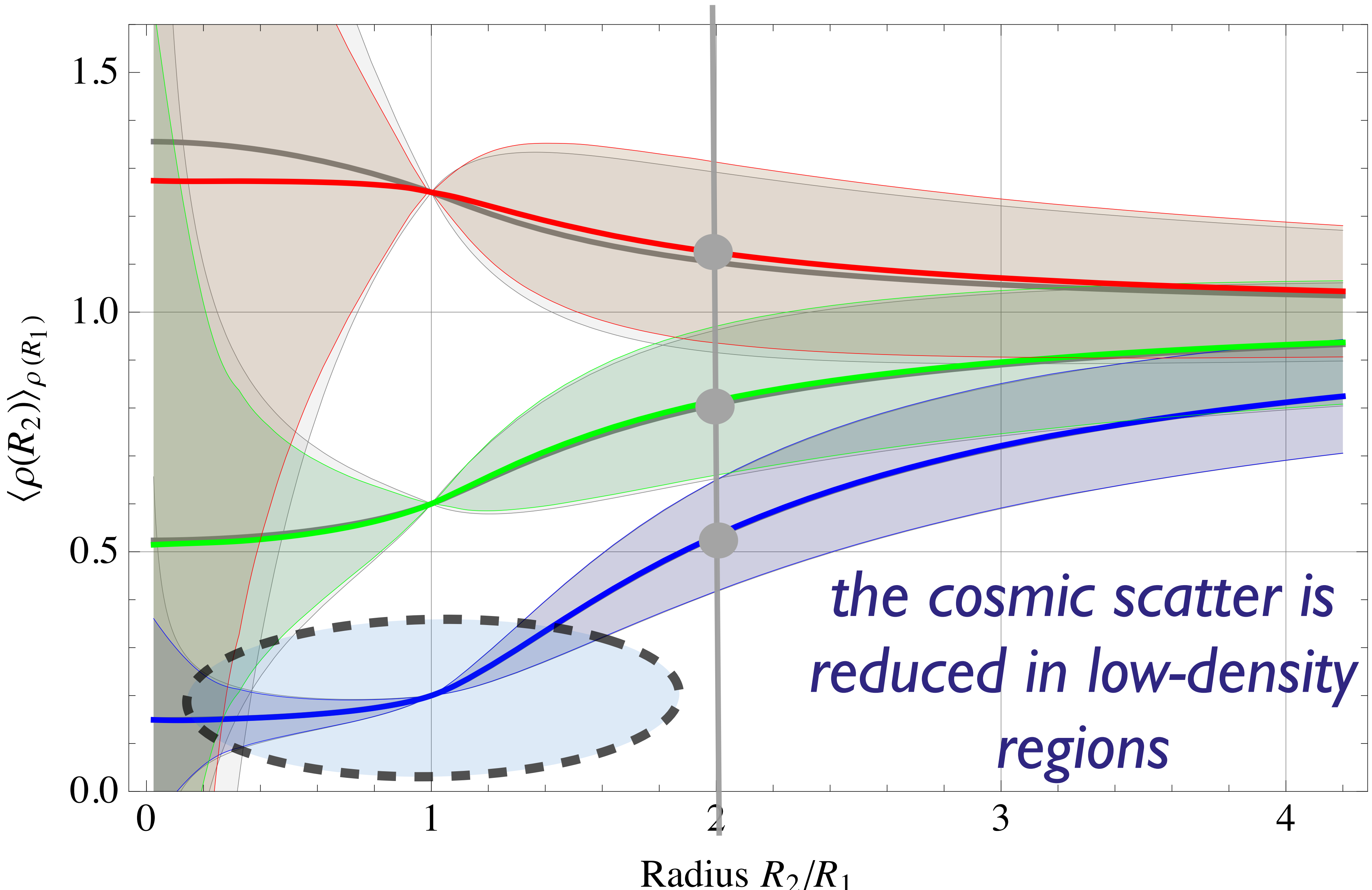

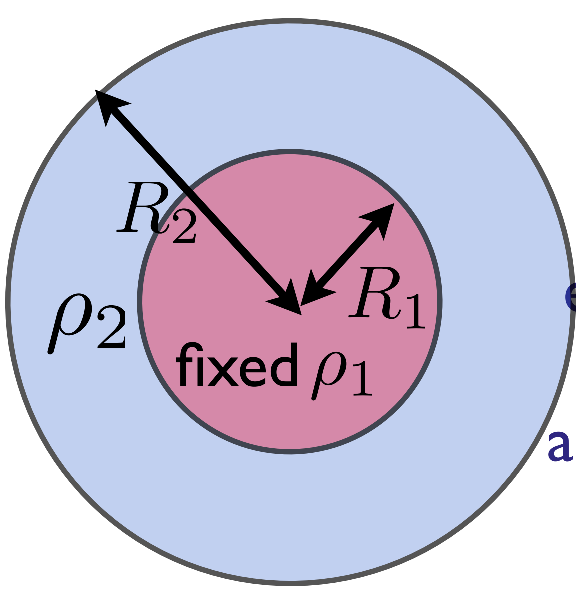

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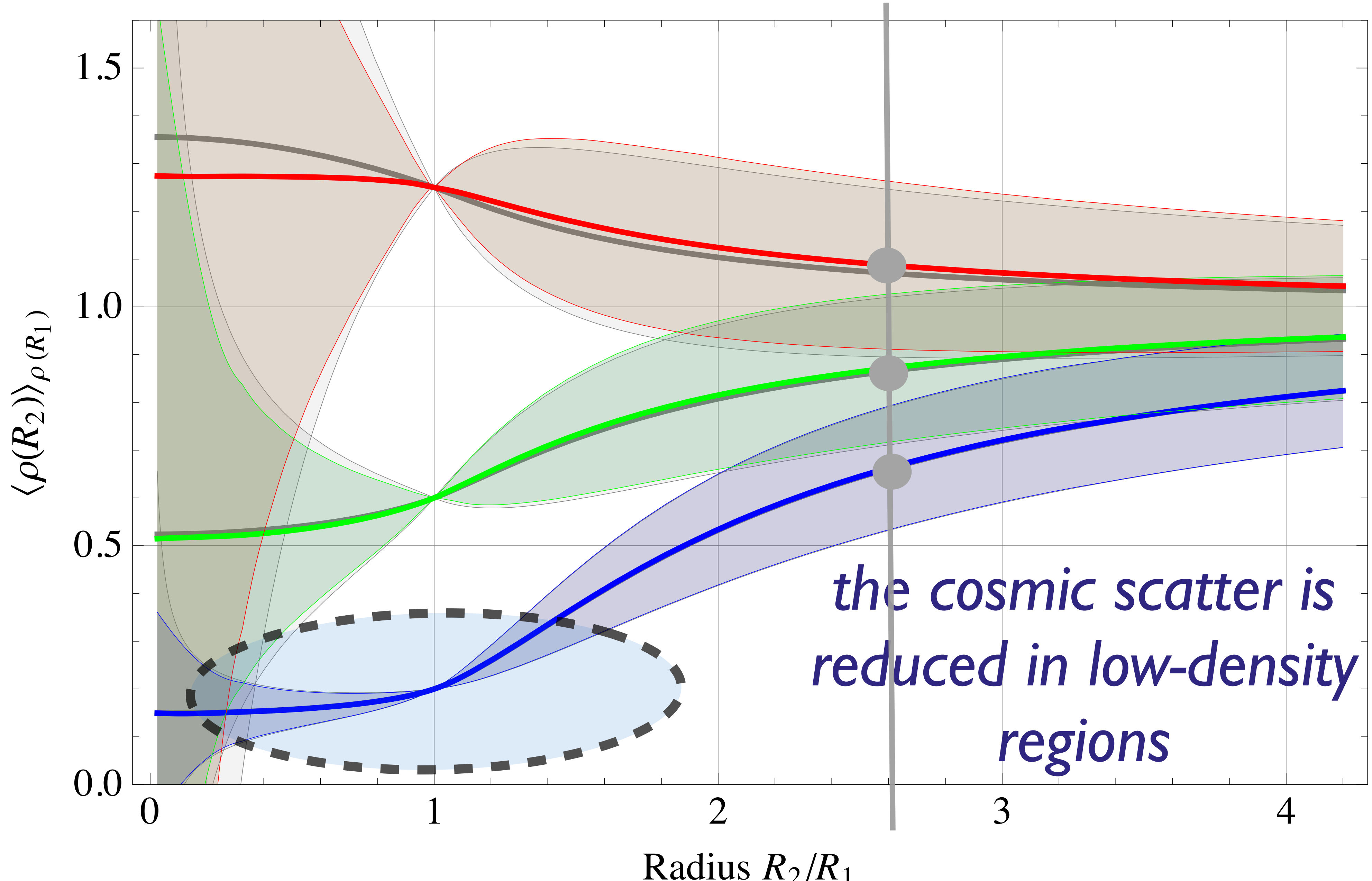

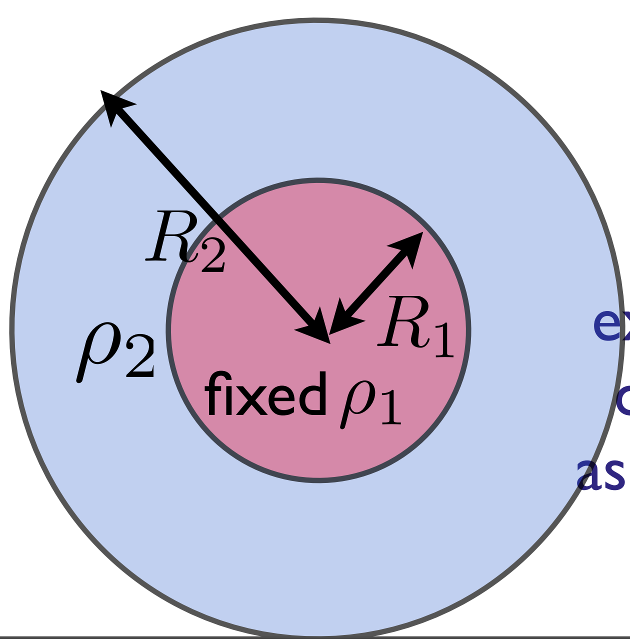

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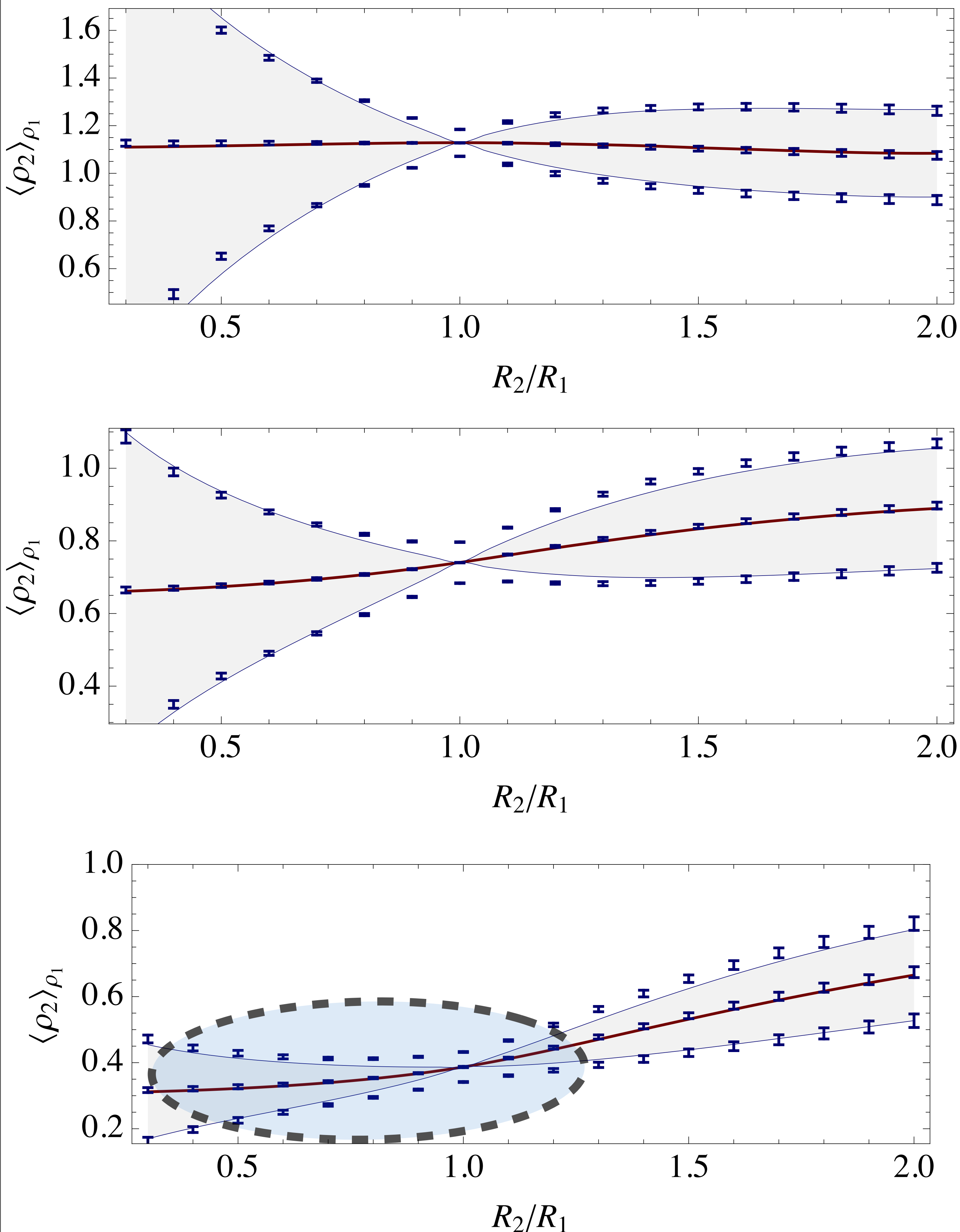
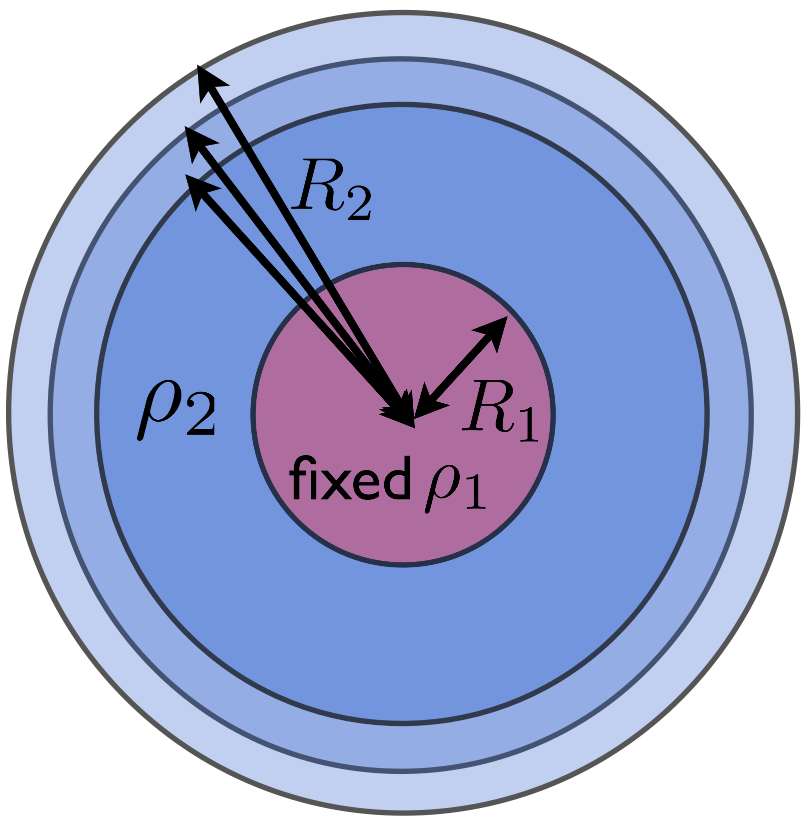


Predictions vs numerical simulations (*expectation+scatter*)



the cosmic scatter is reduced in low-density regions

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fiducial cosmological experiment

Prediction for full joint PDF densities in concentric cells:

$$P(\rho(R_1),\rho(R_2))\ \mathrm{d}\rho(R_1)\ \mathrm{d}\rho(R_2)$$

which is gravity and cosmology-dependent through the linear power spectrum and the dynamics of the spherical collapse.

fiducial cosmological experiment

-power-law power spectrum with index ns (-2.5)

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-we assume that the PDF is well-described by its saddle-point approximation and depends on 2 parameters : **n_s** and **ν**(which parametrizes the spherical collapse, 3/2 here)

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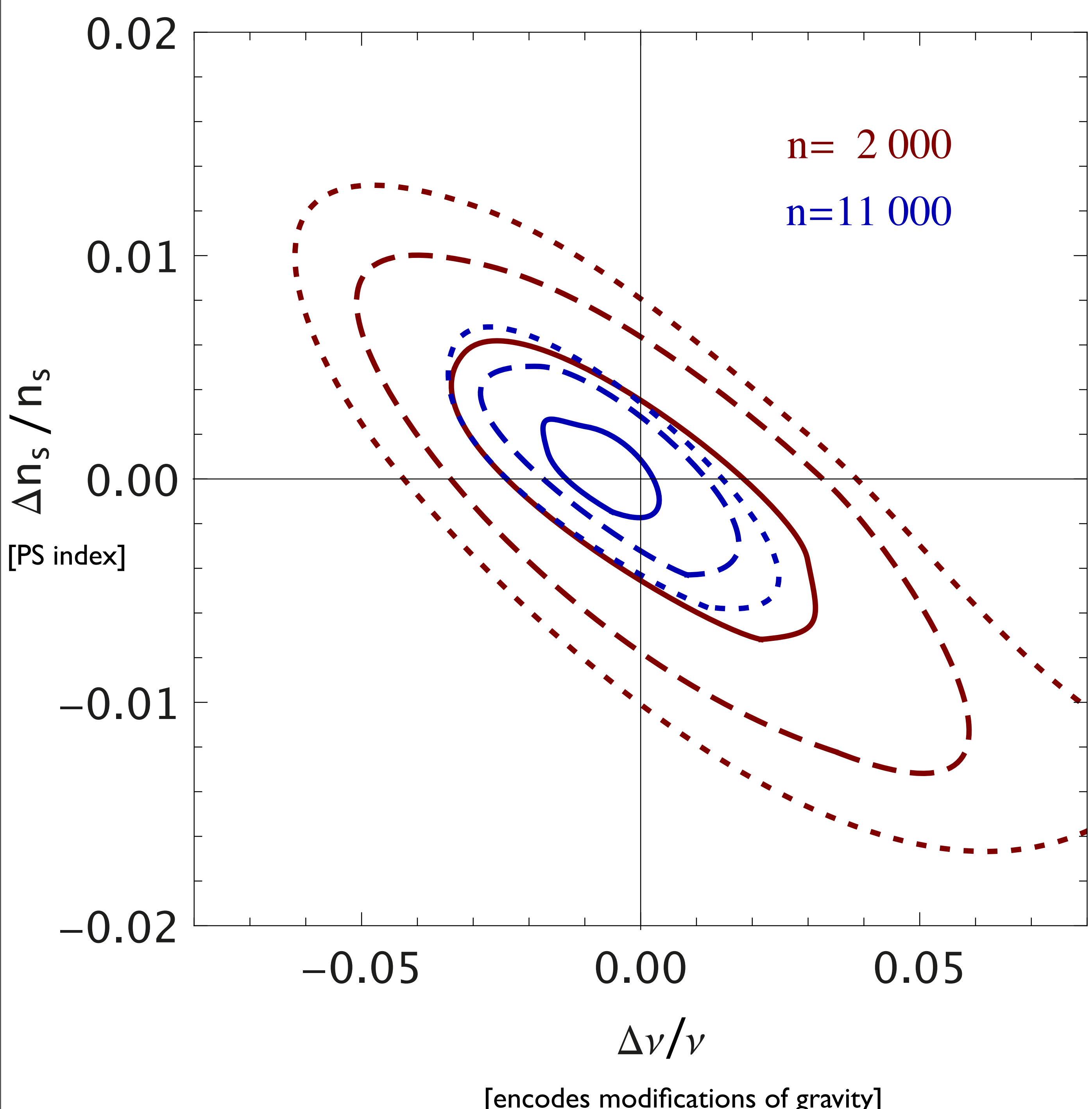

loglikelihood contours of the data at 1, 3 and 5 sigma

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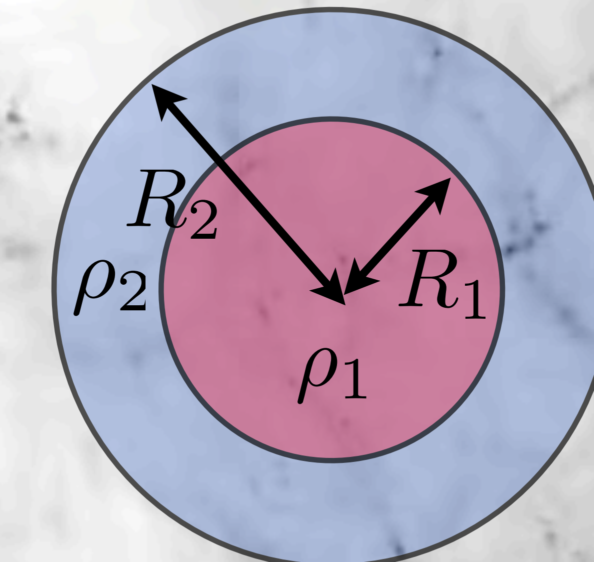
➢ we are able to do the theory of the *slope* of the density field: cosmic scatter is reduced in low-density regions motivating the study of ***void profiles.***

➢ *These calculations can be applied to projected mass maps*



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