

Weak Lensing Measurements

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WL Measurements

- **PSF Modeling**

- Galaxy Shape Measurements (shear)

- Shear Catalog Creation

} OU-Shear

- **2D & 3D Mass Mapping**

- **Peak Counting & High Order Stat**

- **2 Point Correlation function**

- **Power spectra**

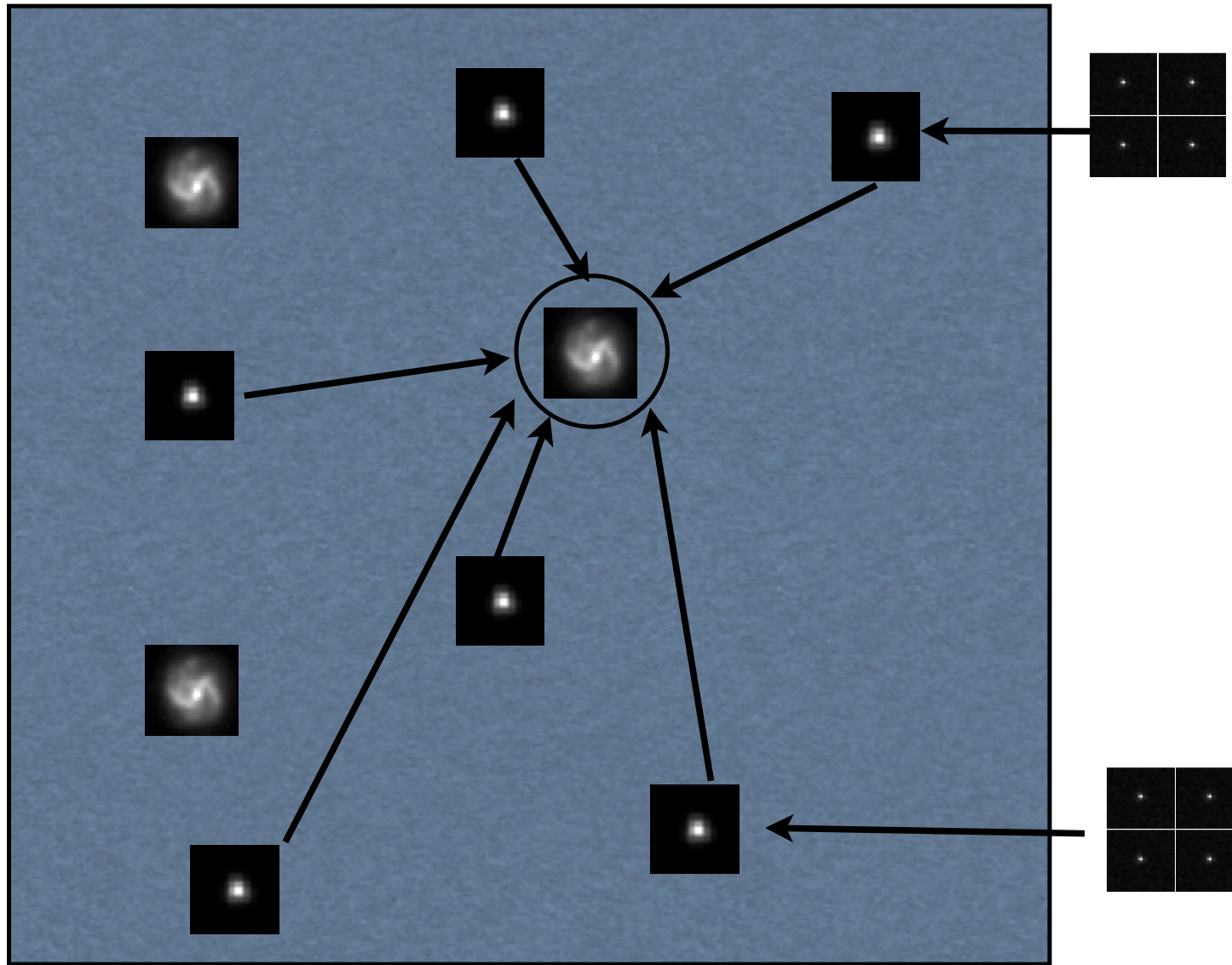
} OU-LE3

=> OULE3 Nice meeting, Dec 18-19, 2013

First evaluation results

<http://adlubitum.oats.inaf.it/meetings/EuclidNice2013>

PSF Representation



Super Resolution in Astronomy

$$Y = HX + N$$



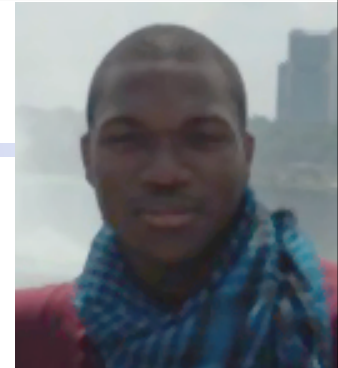
HR image $X = (x_{i,j}), 1 \leq i, j \leq NL$

LR Images $Y_k = (y_{k,i,j}), 1 \leq i, j \leq L$

- Shift-and-add or image stacking
 - ➔ Estimate the centroids (i_k, j_k) for each LR images Y_k
 - ➔ Oversample each image Y_k using a given interpolation method and shift it to match Y_1
 - ➔ Coadd all oversampled registered images

- PSFextractor (PSFext)
$$J(X) = \sum_{k=1}^p \sum_{i=1}^L \sum_{j=1}^L \frac{(y_{k,ij} - f_k \tilde{y}_{k,ij})^2}{\sigma_k^2} + \frac{\|X - X^{(0)}\|_{l_2}^2}{\sigma}$$

Sparse Regularization



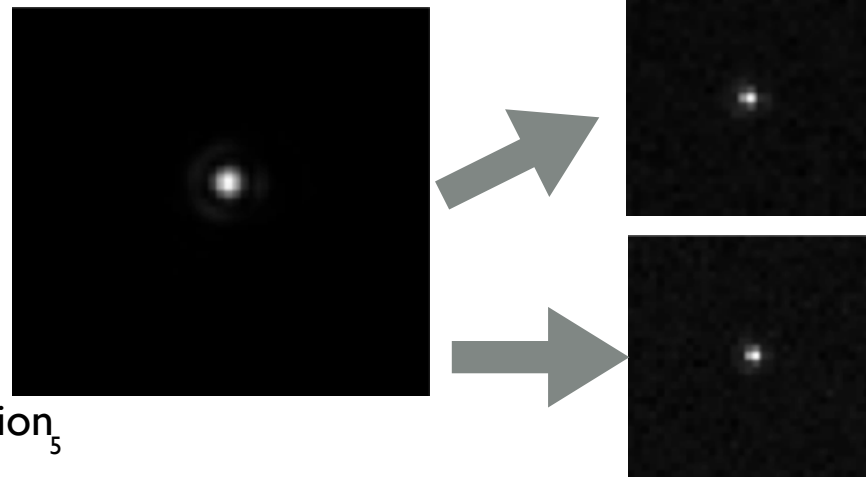
Fred Ngole

$$\left\{ \begin{array}{l} \min_{\Delta_X} \| Y - H(X^{(0)} + \Delta_X) \|_2^2 \\ \text{+ sparsity constraint} \quad \Delta_X = \Phi \alpha \\ X^{(0)} \quad \begin{array}{l} \text{– calculated with shift-and-add} \\ \text{– Registration based on centroids positions} \end{array} \end{array} \right.$$

$$\min_{\Delta_X} \| Y - H(X^{(0)} + \Delta_X) \|_2^2 \quad s.t. \quad \lambda \| \Phi^t \Delta_X \|_1$$

Experiments

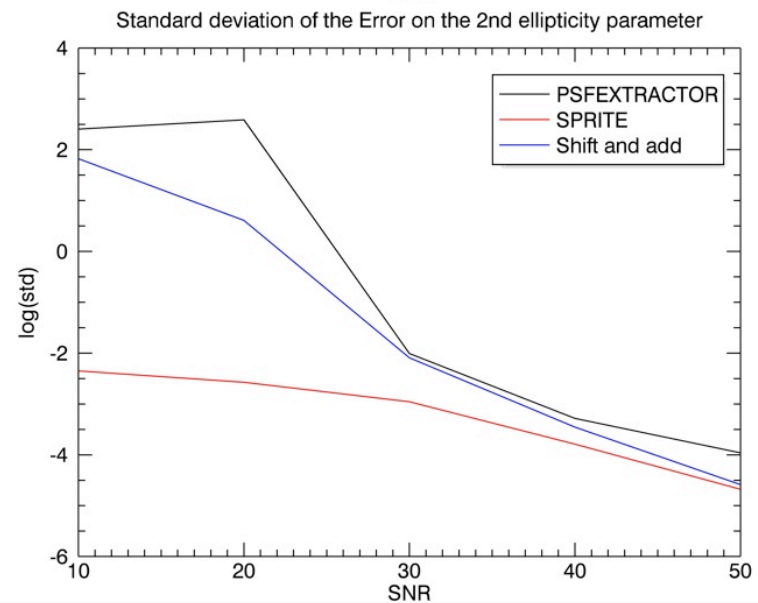
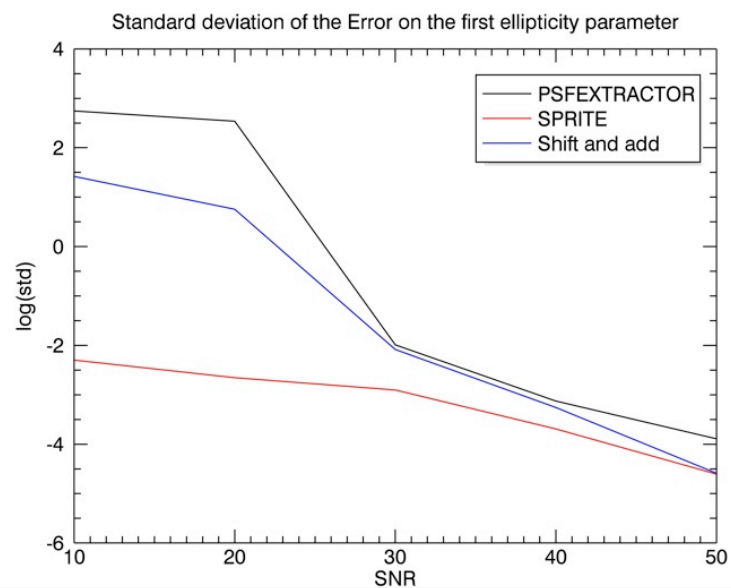
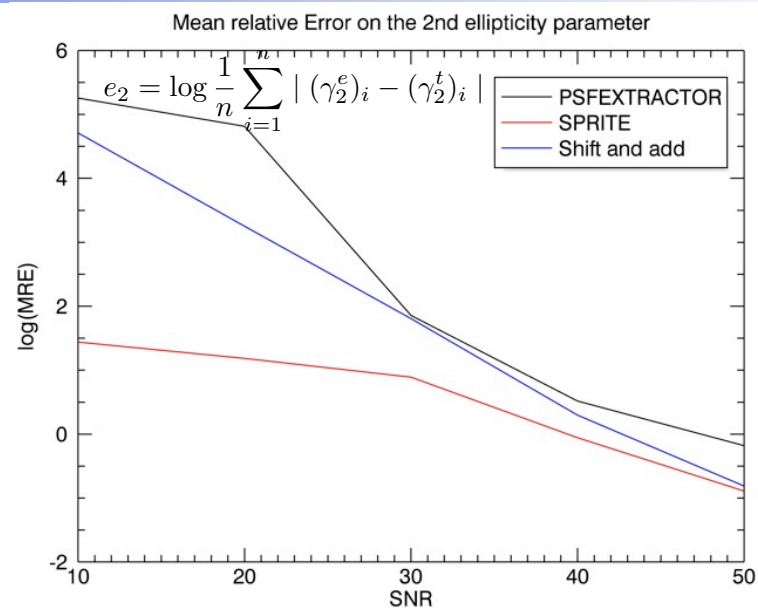
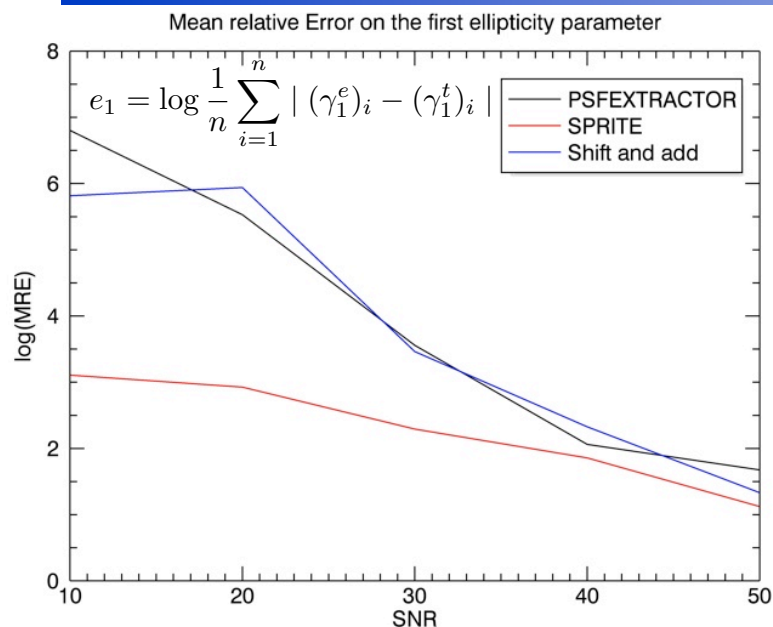
- 150 Zemax PSF at 12 x Euclid Resolution
- For each PSF, 4 randomly shifted and noisy PSF at Euclid resolution

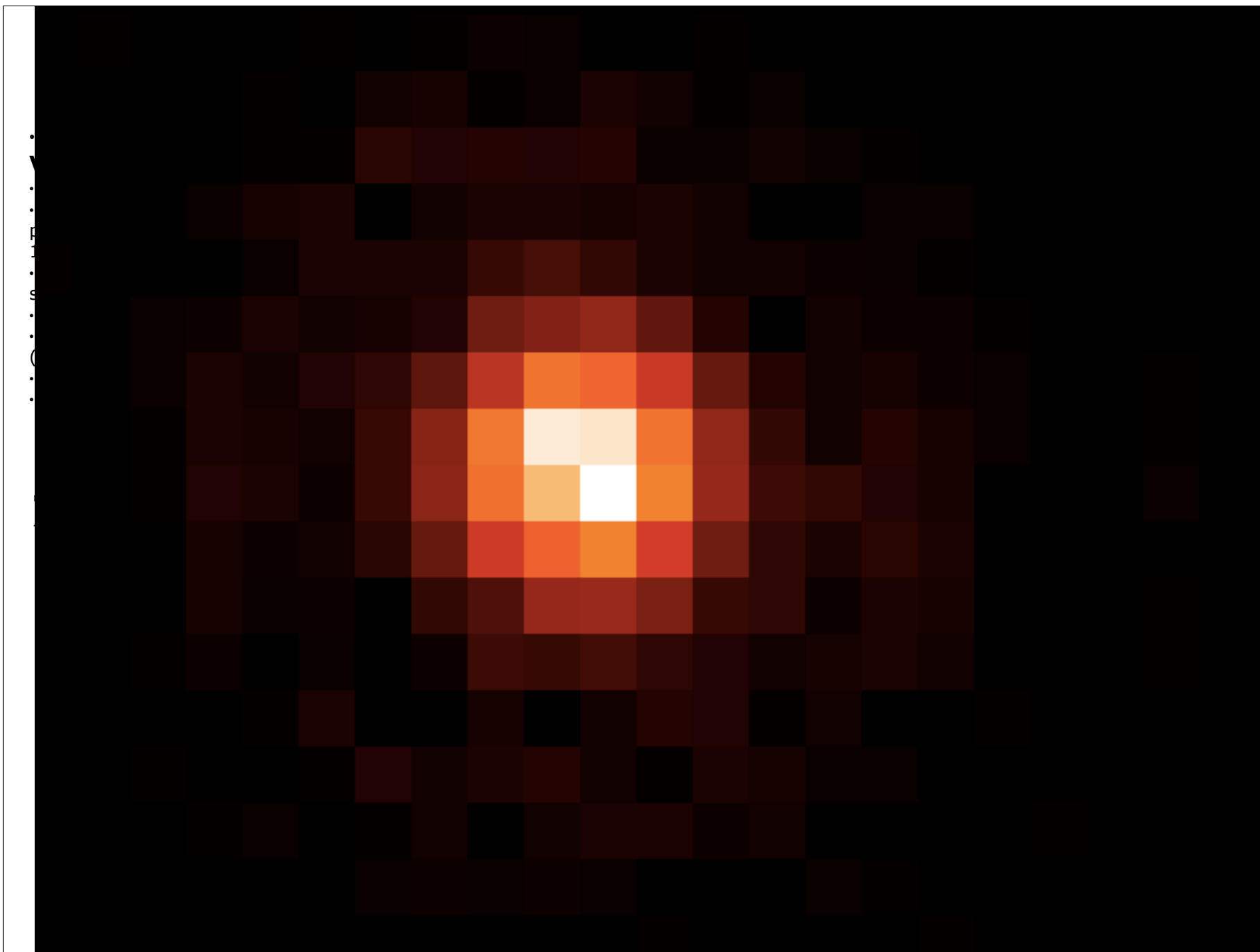


GOAL: PSF modeling at twice Euclid resolution₅

Numerical Experiment

==> Goal: Reconstruction these PSF at 2 x Euclid Resolution from 4 subsample noisy images.





Mass Mapping

The shear is related to the convergence through the relation:

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\boldsymbol{\theta}' \mathcal{D}(\boldsymbol{\theta} - \boldsymbol{\theta}') \kappa(\boldsymbol{\theta}') \quad \text{with} \quad \mathcal{D}(\boldsymbol{\theta}) \equiv \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1\theta_2}{|\boldsymbol{\theta}|^4} = -\frac{1}{(\theta_1 - i\theta_2)^2}$$

The convergence is related to the 3D density contrast

$$\kappa(\boldsymbol{\theta}, w) = \frac{3H_0^2\Omega_M}{2c^2} \int_0^w dw' \frac{f_K(w') f_K(w - w')}{f_K(w)} \frac{\delta[f_K(w')\boldsymbol{\theta}, w']}{a(w')} \quad \text{with} \quad \delta(\boldsymbol{r}) \equiv \rho(\boldsymbol{r})/\bar{\rho} - 1$$

where $\bar{\rho}$ is the mean density of the Universe, H_0 is the Hubble parameter, Ω_M is the matter density parameter, c is the speed of light, $a(w)$ is the scale parameter evaluated at comoving distance w , and

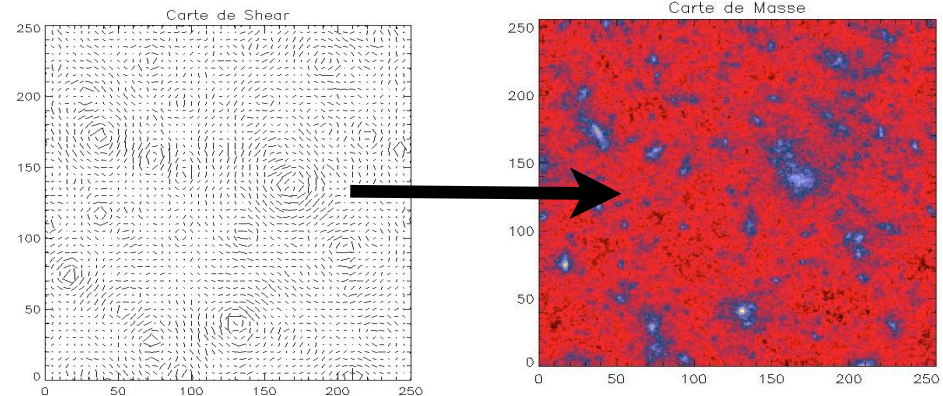
$$f_K(w) = \begin{cases} K^{-1/2} \sin(K^{1/2}w), & K > 0 \\ w, & K = 0 \\ (-K)^{-1/2} \sinh([-K]^{1/2}w) & K < 0 \end{cases} ,$$

gives the comoving angular diameter distance as a function of the comoving distance and the curvature, K , of the Universe.

2D Mass Mapping

Aperture mass maps or Convergence (kappa) maps ?

$$\left\{ \begin{array}{l} M_{ap}(\boldsymbol{\theta}) = \int d^2\boldsymbol{\vartheta} \, \kappa(\boldsymbol{\vartheta}) U(|\boldsymbol{\vartheta}|) \\ M_{ap}(\boldsymbol{\theta}) = \int d^2\boldsymbol{\vartheta} \, \gamma_t(\boldsymbol{\vartheta}) Q(|\boldsymbol{\vartheta}|) \\ Q(\vartheta) \equiv \frac{2}{\vartheta^2} \int_0^\vartheta \vartheta' U(\vartheta') d\vartheta' - U(\vartheta) \end{array} \right. \quad \left\{ \begin{array}{l} \gamma(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\boldsymbol{\theta}' \mathcal{D}(\boldsymbol{\theta} - \boldsymbol{\theta}') \kappa(\boldsymbol{\theta}') \\ \mathcal{W}_j(x, y) = \int_{-\infty}^{+\infty} \kappa(x, y) \psi_j(x, y) dx dy \end{array} \right.$$



⇒ Wavelets filters are formally indentical to Mass aperture

S. Pires, A. Leonard, J.-L. Starck, ["Cosmological Parameters Constraint from Weak Lensing Data"](#), **MNRAS**, 423, pp 983-992, 2012.

A. Leonard, S. Pires, J.-L. Starck, ["Fast Calculation of the Weak Lensing Aperture Mass Statistic"](#), **MNRAS**, 423, pp 3405-3412, 2012.

A. Leonard, J.-L. Starck, S. Pires, F.-X Dupe, [Exploring the Components of the Universe Through Higher-Order Weak Lensing Statistics](#), Open Questions in Cosmology, Gonzalo J. Olmo (Ed.), ISBN: 978-953-51-0880-1, InTech, DOI: 10.5772/51871, 2012.



S. Pires

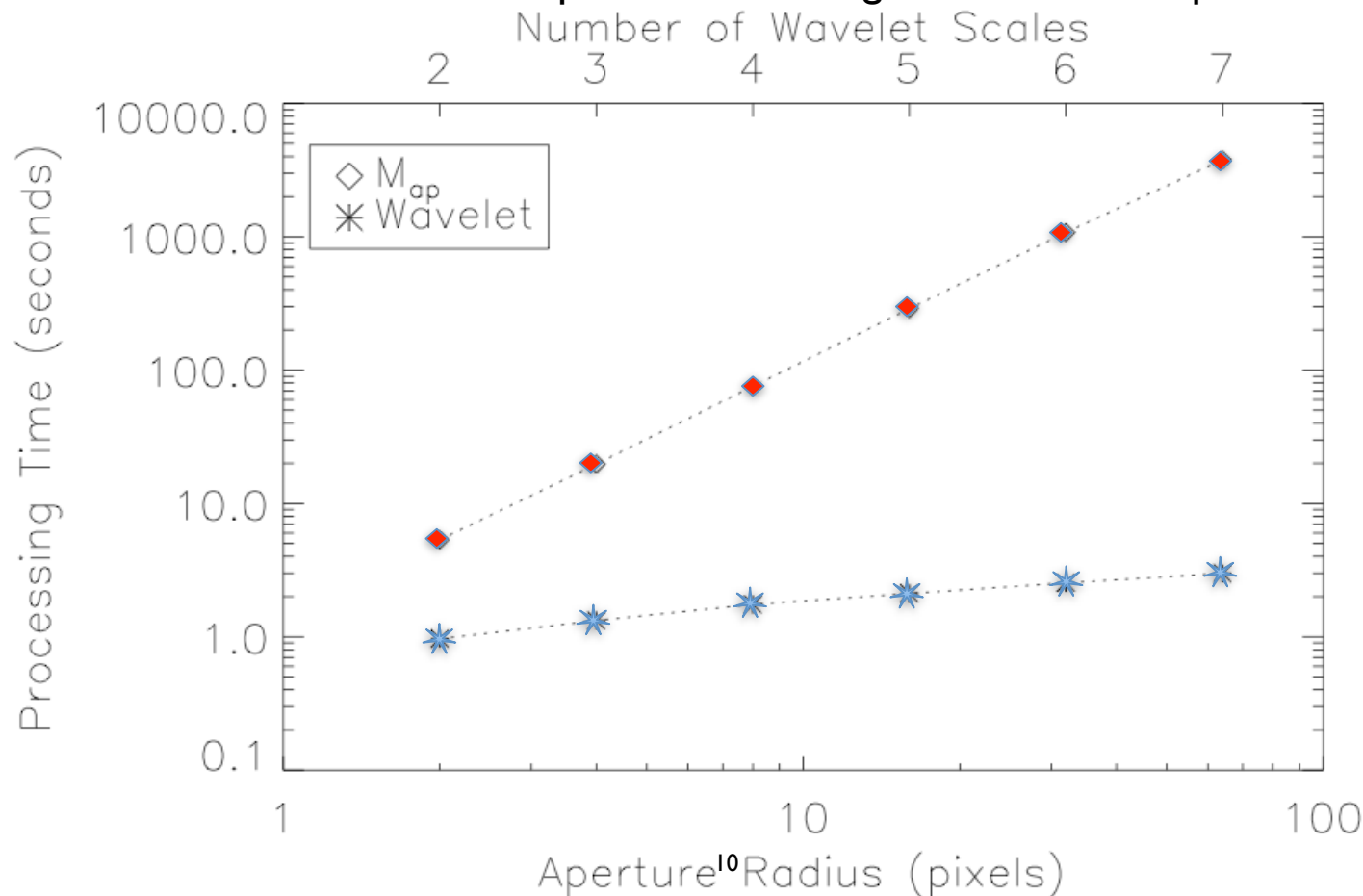
Mass Mapping

but wavelets presents many advantages:

- compensated and **compact** support filters
- **fast** calculation: $N \log N$ instead of N^2
- **all scales** processed in one step.
- **reconstruction** is possible ==> image restoration for peak counting



A. Leonard



3D Mass Mapping

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\boldsymbol{\theta}' \mathcal{D}(\boldsymbol{\theta} - \boldsymbol{\theta}') \kappa(\boldsymbol{\theta}')$$

Kappa (or convergence) is a dimensionless surface mass density of the lens

$$\kappa(\boldsymbol{\theta}, w) = \frac{3H_0^2\Omega_M}{2c^2} \int_0^w dw' \frac{f_K(w')f_K(w-w')}{f_K(w)} \frac{\delta[f_K(w')\boldsymbol{\theta}, w']}{a(w')},$$

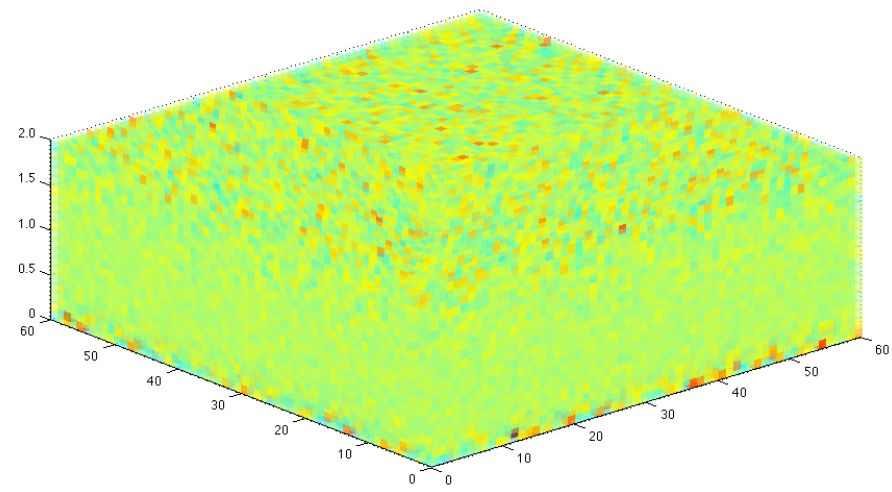
f_K is the angular diameter distance, which is a function of the comoving radial distance r and the curvature K .

$$\boldsymbol{\gamma} = \mathbf{P}_{\gamma\kappa} \boldsymbol{\kappa} + \mathbf{n}_\gamma,$$

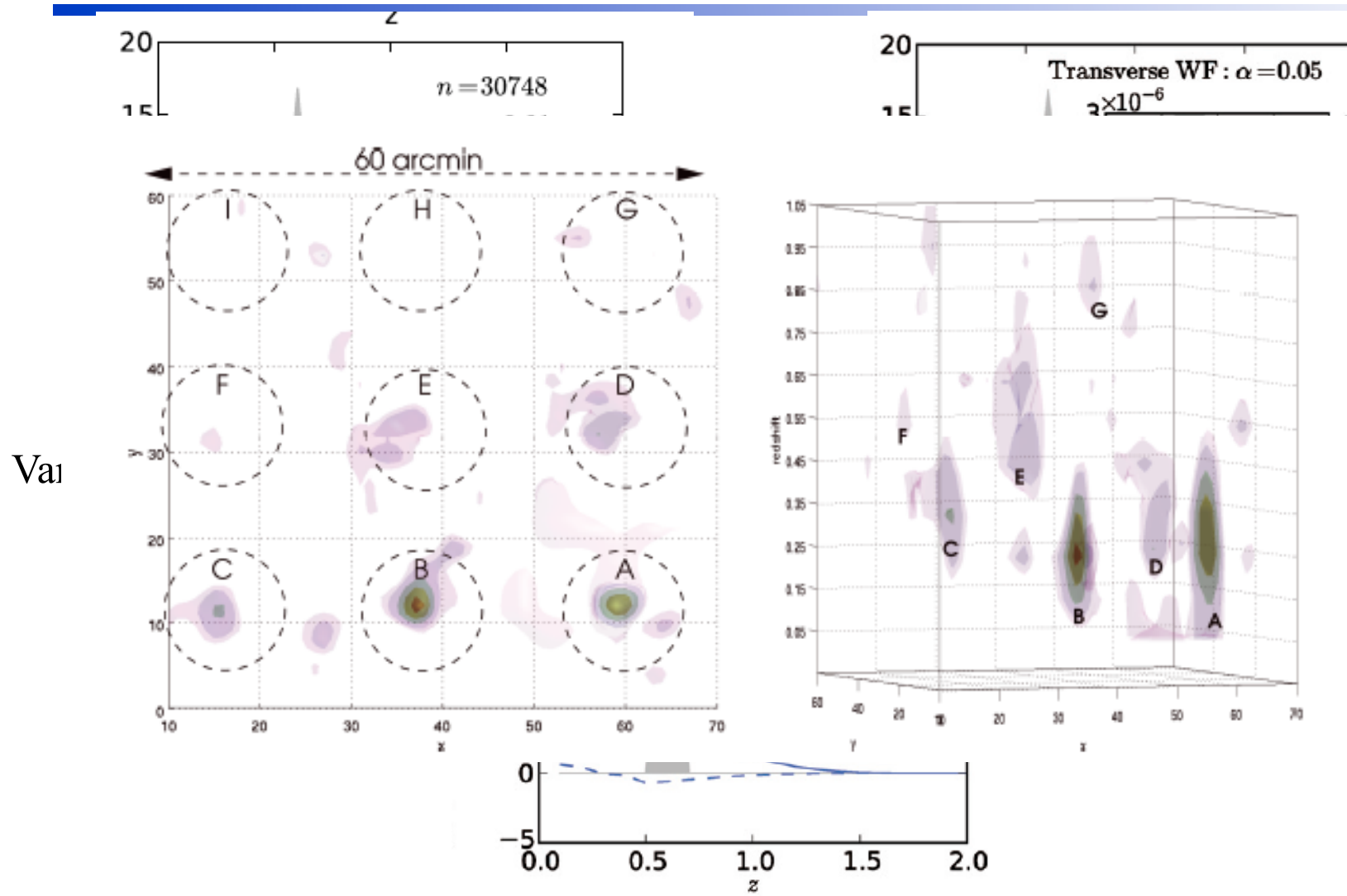
$$\kappa = Q\delta + n$$

$$\boldsymbol{\gamma} = \mathbf{R}\boldsymbol{\delta} + \mathbf{n}$$

- Galaxies are not intrinsically circular: intrinsic ellipticity ~ 0.2 - 0.3 ; gravitational shear ~ 0.02
- Reconstructions require knowledge of distances to galaxies



3D Reconstruction: Linear Approach



Simon et al. 2009, MNRAS, 399, 48



A. Leonard

Full 3D Weak Lensing

A. Leonard, F.X. Dupe, and J.-L. Starck, "[A Compressed Sensing Approach to 3D Weak Lensing](#)", *Astronomy and Astrophysics*, 539, A85, 2012.

A. Leonard, F. Lanusse, J.-L. Starck, **GLIMPSE**: Accurate 3D weak lensing reconstruction using sparsity, *Astronomy and Astrophysics*, submitted, 2013

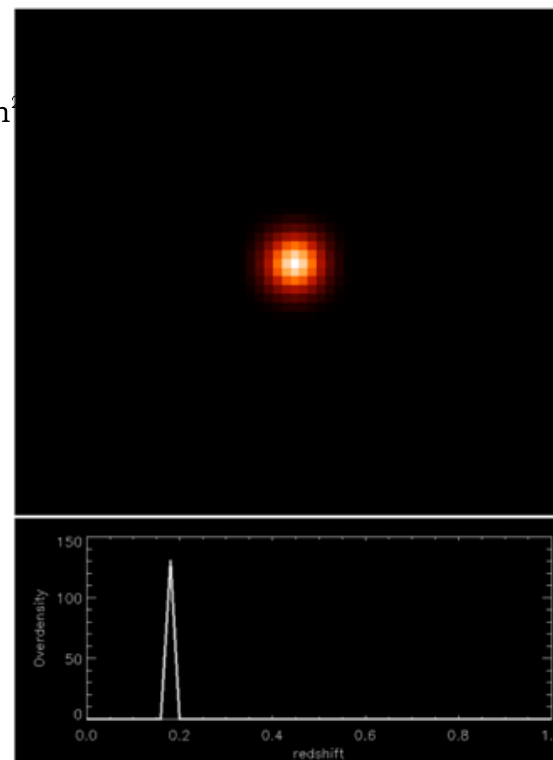
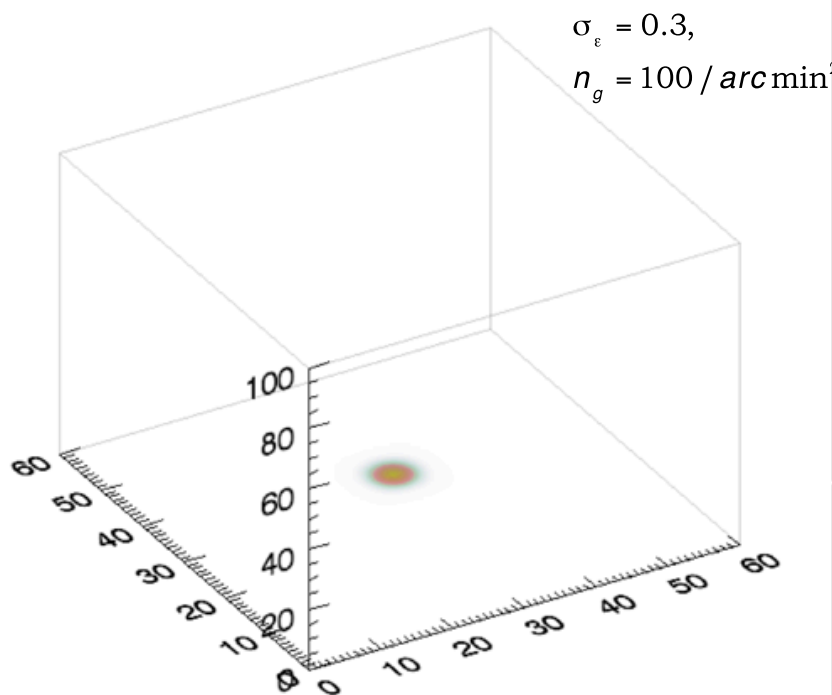


F. Lanusse

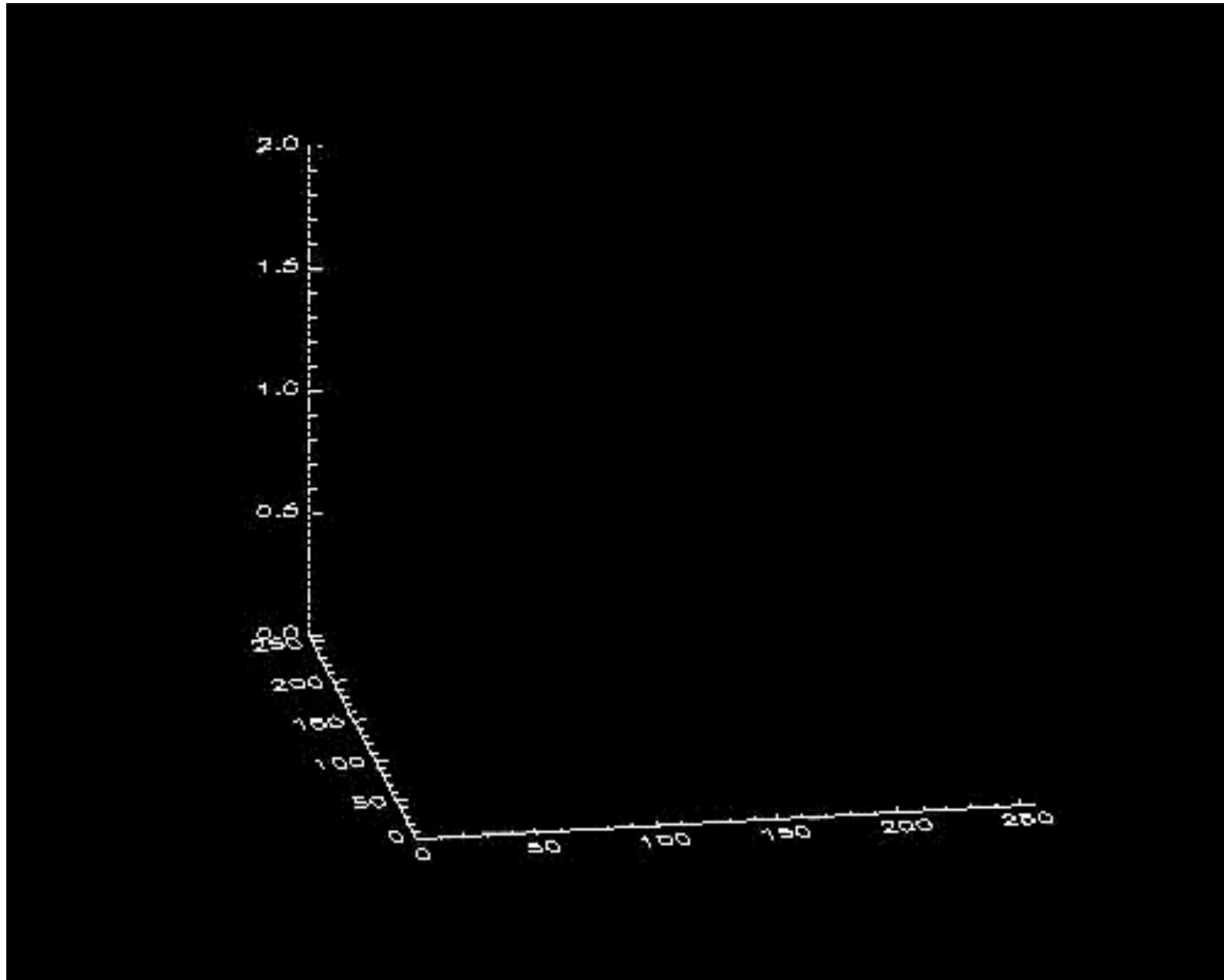
$$\min_{\alpha} \|\alpha\|_1 \quad s.t. \quad \frac{1}{2} \|\gamma - R\Phi\alpha\|_{\Sigma^{-1}}^2 \leq \epsilon$$

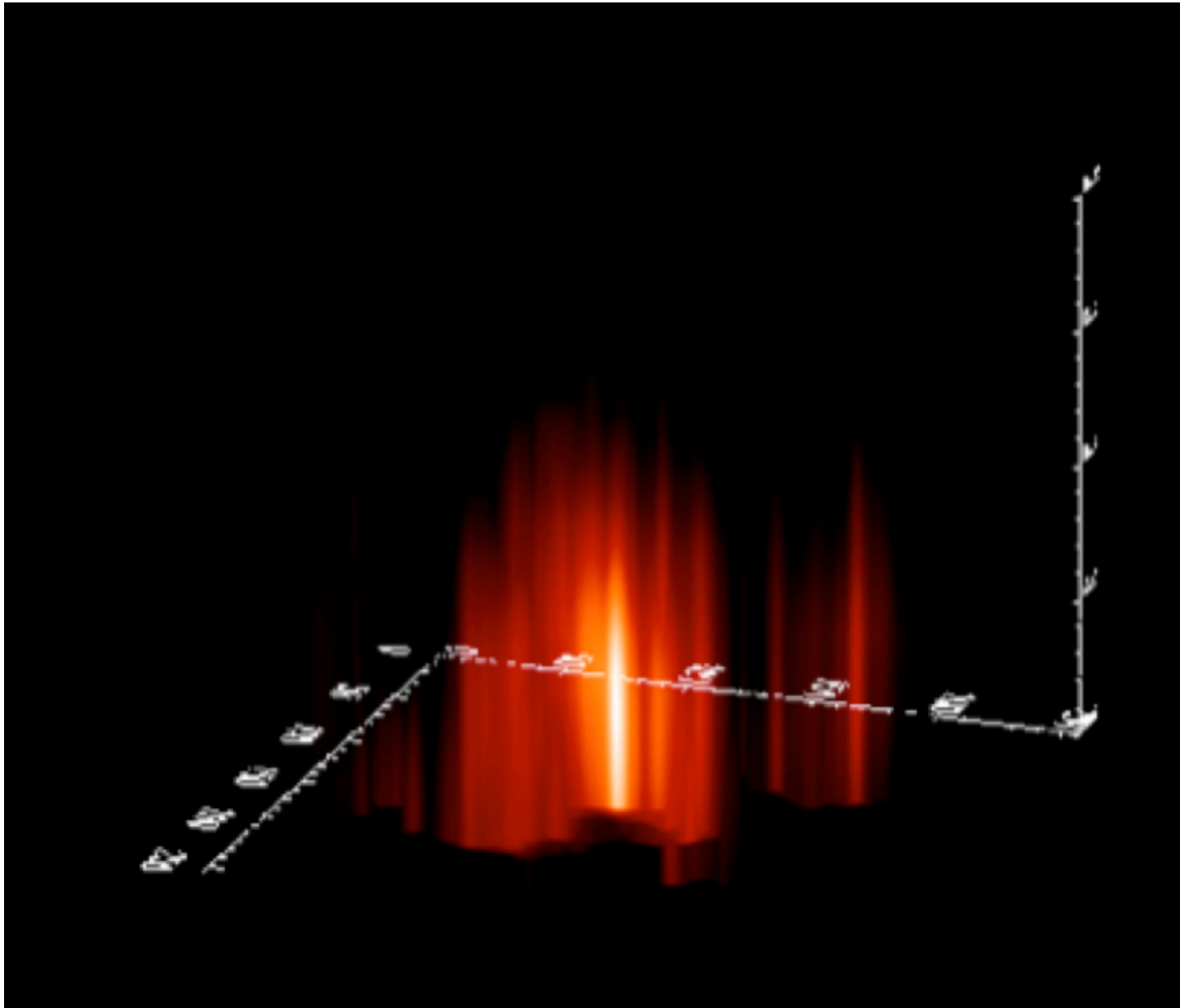
$$\delta = \Phi\alpha \quad \Phi = \text{2D Wavelet Transform on each redshift bin}$$

Example of the reconstruction of a cluster at redshift $z=0.2$ using 5 times more reconstruction bin:



<http://cosmostat.org/glimpse.html>





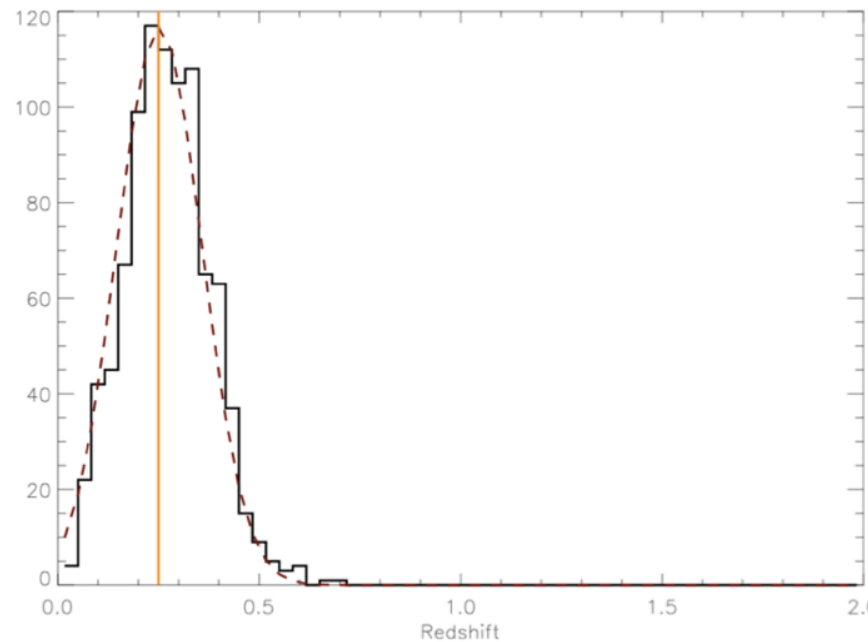
Mass & Redshift Estimation

Single halo simulations

- One NFW profile at the center of a 60x60 arcmin field
- Noise and redshift errors corresponding to an Euclid-like survey
- Mass varying between 3.10^{13} and $1.10^{15} h^{-1} M_{\odot}$
- Redshifts between 0.05 and 1.55

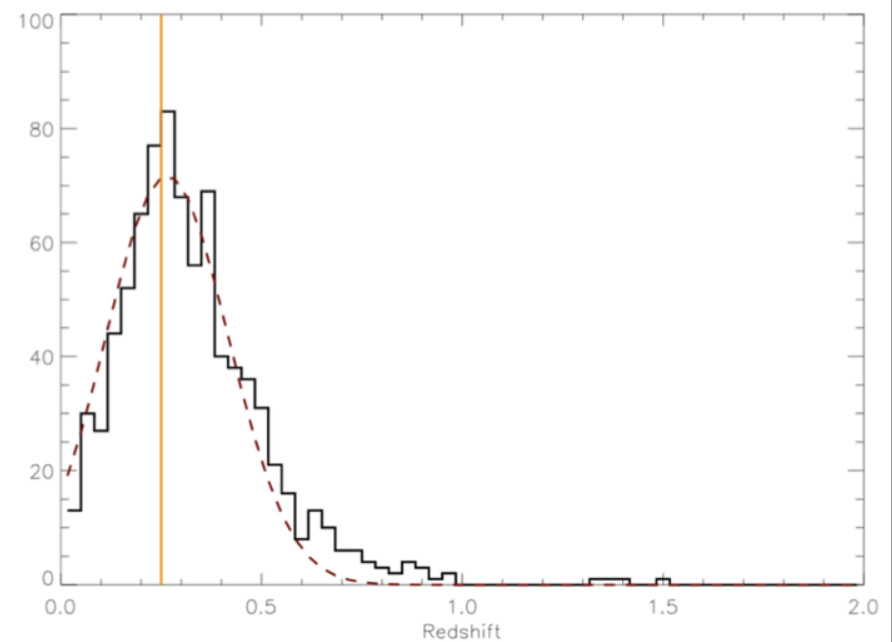
We ran 1000 noise realisations on each of the 96 fields.

$$m_{vir} = 8.10^{14} h^{-1} M_{\odot}$$



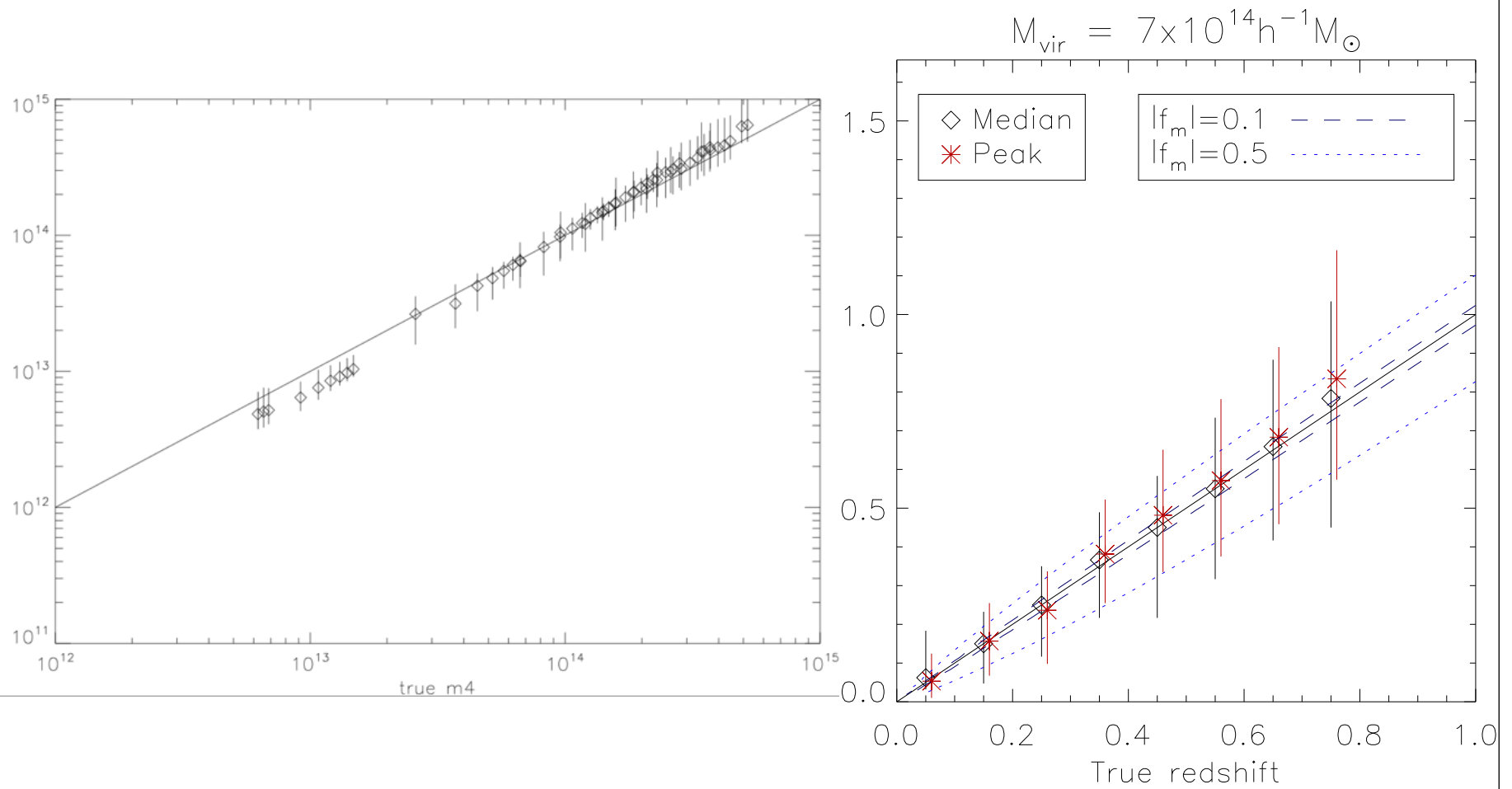
$$\sigma_z = 0.1$$

$$m_{vir} = 4.10^{14} h^{-1} M_{\odot}$$



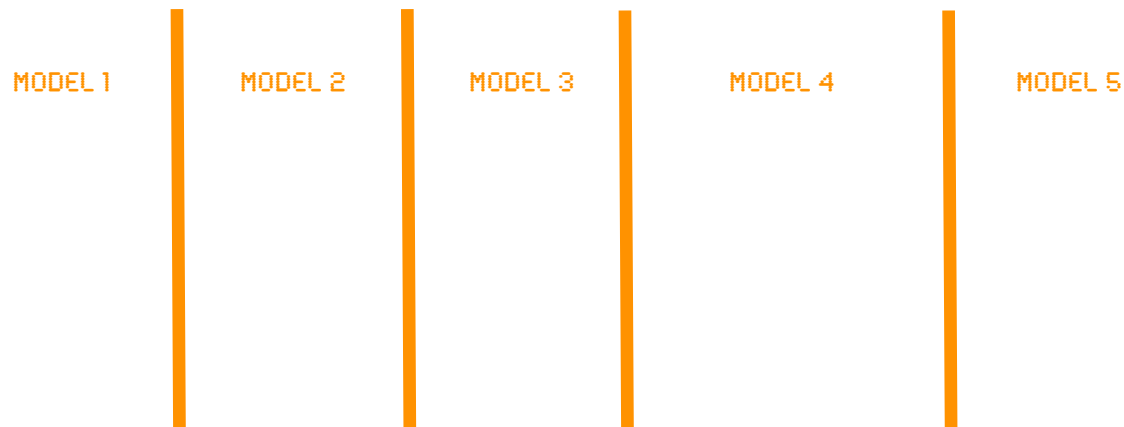
$$\sigma_z = 0.15$$

Mass Estimation



Conclusions

- **PFS Modeling using Sparsity (SPRITE method) seems to be very efficient.**
- **2D map: Aperture mass map = wavelets, but wavelets calculation is between 10 and 1000 times faster.**
- **3D map: GLIMPSE solves many problems related to linear methods:**
Redshift bias, Smearing, Damping, Resolution limited by data
- **Wavelet Denoising + Wavelet Peak Counting is the most efficient statistical tool to discriminate Cosmological Models.**



WAVELET PEAK COUNTING ON MRLENS FILTRED MAPS (AT SCALE OF ABOUT 1 ARCMIN)